Diffuse radio sources on Mpc scales detected in the outskirts of galaxy clusters: synchrotron radiation emitted by ~GeV electrons accelerated at structure formation shocks via DSA (Fermi I) process.
Characteristics of collisionless shocks depend on B field direction

**Quasi-parallel** = Q-par

\[ \theta_{Bn} < 45^\circ \]

**Quasi-perpendicular** = Q-perp

\[ \theta_{Bn} > 45^\circ \]

Self generated waves

Hybrid simulations by Caprioli & Spitkovsky 2014
### Properties of Astrophysical Plasmas

<table>
<thead>
<tr>
<th></th>
<th>solar wind (IPM)</th>
<th>ISM</th>
<th>ICM</th>
<th>solar flare</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_H$ ($\text{cm}^{-3}$)</td>
<td>5</td>
<td>0.1</td>
<td>$10^4$</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$T$ ($^\circ\text{K}$)</td>
<td>$10^5$</td>
<td>$10^4$</td>
<td>$5 \times 10^7$</td>
<td>$10^5$-$10^6$</td>
</tr>
<tr>
<td>$B$ ($\mu\text{G}$)</td>
<td>50</td>
<td>5</td>
<td>1</td>
<td>$10^8$</td>
</tr>
<tr>
<td>$c_s$ ($\text{km/s}$)</td>
<td>50</td>
<td>15</td>
<td>1000</td>
<td>50-150</td>
</tr>
<tr>
<td>$v_A$ ($\text{km/s}$)</td>
<td>40</td>
<td>30</td>
<td>180</td>
<td>2000</td>
</tr>
<tr>
<td>$\beta_p = P_g/P_B$</td>
<td>1.6</td>
<td>0.3</td>
<td>40</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha_p = \omega_p/\Omega_e$</td>
<td>140</td>
<td>200</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>$u_s$ ($\text{km/s}$)</td>
<td>500</td>
<td>3000</td>
<td>2000</td>
<td>-</td>
</tr>
<tr>
<td>$M_s = u_s/c_s$</td>
<td>10</td>
<td>200</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$M_A = u_s/v_A$</td>
<td>13</td>
<td>100</td>
<td>11</td>
<td>-</td>
</tr>
</tbody>
</table>

ICM (cluster shocks) vs ISM (SNR shocks)

higher $\beta_p$: B pressure is dynamically less important in ICM

lower $\alpha_p$: wave-part. interactions and stochastic acceleration more significant in ICM

particle acceleration at collisionless shocks depend on $M_s$, $M_A$, $\theta_{Bn}$, $\beta_p$, $\alpha_p$
Particle Acceleration Processes at collisionless shocks

(1) Diffusive Shock Acceleration (DSA): Fermi 1st order process
- effective at quasi-parallel (Q-par) shocks
- scattering off MHD waves in the upstream and downstream region

(2) Shock Drift Acceleration (SDA)
- effective at quasi-perpendicular (Q-perp) shocks
- drifting along the convective E field (grad B) at the shock front

(3) Shock Surfing Acceleration (SSA)
- effective at quasi-perpendicular (Q-perp) shocks
- reflected by shock potential, scattered by upstream waves
- moving along the convective E field, while being trapped at the shock foot

(4) Turbulent Acceleration (TA): Fermi 2nd order, stochastic acceleration
- much less efficient than Fermi I
- could be important in turbulent plasmas

(5) Turbulent Reconnection (TR) of Magnetic Fields

Essential ingredients: $\vec{B}_0$ & $\partial \vec{B}$
**DSA: Fermi first order process**

MHD waves in a converging flow act as converging mirrors

→ particles are scattered & isotropized by waves in local fluid frame

→ cross the shock many times

\[ \Delta p \sim \frac{u_1 - u_2}{v} \text{ at each shock crossing} \]

**Test-particle limit solution**

\[ f_{test}(p) \propto p^{-q_{test}} : \text{power-law} \]

\[ q_{test} = \frac{3u_1}{(u_1 - u_2)} = \frac{4M_s^2}{M_s^2 - 1} \]

**Radio synchrotron emission**

\[ j_v \propto v^{-\alpha} \]

\[ \alpha = \frac{q - 3}{2} = \frac{M_s^2 + 3}{2(M_s^2 - 1)} \]
Evolution of an electron undergoing multiple SDA

Reflection of some electrons due to Magnetic mirror at shock
- Induce temperature anisotropy in the upstream
- Firehose instability excites waves
- Reflected electrons are scattered back to the shock downstream by the waves
- Undergo multiple SDA cycles

\[ T_{||,e} > T_{\perp,e} \]

Multiple SDA for electrons
- Suprathermal tail
- Pre-acceleration for DSA
- Injected to Fermi I
Stochastic energy gain in collisions with MHD/plasma waves (head-on collisions are more frequent than over-taking collisions) \( \Rightarrow \) 2\(^{nd}\) order in energy gain (slow and inefficient)

CR particles (protons + electrons) could be accelerated via Fermi II acceleration by turbulence in ICM.
- **Alfvén and slow modes** become anisotropic at small scales, so scattering by Alfvénic turbulence becomes inefficient. (Cho & Lazarian 2003)
- But **fast mode** remains isotropic at small scales, so **Transit Time Damping (TTD)** with fast modes is dominant in ICM (Brunetti & Lazarian 2007)
cluster CIZAJ2242.8+5301

Sausage Relic

Shock signatures
1. Spectral steepening behind the shock
2. Elongated morphology
3. High polarization up to 50% (B field compression)

\[ F_v \propto v^{-\alpha} \]

\[ M_{radio}^2 = \frac{(3 + 2\alpha_{sh})}{(2\alpha_{sh} - 1)} \]

\[ M_{radio} \approx 4.6 \]

but \[ M_X \approx 2.7 \text{ from} \]

\[ \frac{T_2}{T_1} = \frac{(M_X^2 + 3)(5M_X^2 - 1)}{16M_X^2} \]
cluster 1RXS J060303.3  Toothbrush Relic

\[ M_{\text{radio}}^2 = \frac{(3 + 2\alpha_{sh})}{(2\alpha_{sh} - 1)} \Rightarrow M_{\text{radio}} \approx 2.8 \quad \text{but} \quad M_x \approx 1.5 \]
Reacceleration Model for Formation of Giant Radio Relics

CR electrons in lobes of AGN jet

\[ \sim \text{Gyr ago} \]

Cooled Fossil electrons with \( \gamma_e < 300 \)

\[ f_{\text{pre}}(p) = f_0 \cdot p^{-s} \exp \left[ -\left( \frac{p}{p_{e,c}} \right)^2 \right] \]

provide seed CR electrons to DSA

\( M_s \approx 3.0 \)
**DSA simulations in test-particle limit**

in a co-expanding frame which expands with 1D spherical shock.

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \tilde{\nu})}{\partial x} = -\frac{2}{a} \tilde{\rho} \tilde{\nu}
\]

ordinary gasdynamic Eqs (high beta)

\[
\frac{\partial (\tilde{\rho} \tilde{\nu})}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \tilde{\nu}^2 + \tilde{P}_g)}{\partial x} = -\frac{2}{a} \tilde{\rho} \tilde{\nu}^2 - \frac{\dot{a}}{a} \tilde{\rho} \tilde{\nu} - \ddot{a} \tilde{\rho}
\]

\[
\frac{\partial (\tilde{\rho} \tilde{e}_g)}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \tilde{e}_g \tilde{\nu} + \tilde{P}_g \tilde{\nu})}{\partial x} = -\frac{2}{a} (\tilde{\rho} \tilde{e}_g \tilde{\nu} + \tilde{P}_g \tilde{\nu}) - 2 \frac{\dot{a}}{a} \tilde{\rho} \tilde{e}_g - \ddot{a} \tilde{\rho} \tilde{\nu} - \tilde{L}(x,t)
\]

\[x = r / a : \text{co-moving coordinate, } a = \text{expansion factor}\]

**CR transport Equation for electron distribution function**

\[
\frac{\partial g_e}{\partial t} + u \frac{\partial g_e}{\partial r} = \frac{1}{3r^2} \frac{\partial (r^2 u)}{\partial r} \left( \frac{\partial g_e}{\partial y} - 4g_e \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \kappa(r, p) \frac{\partial g_e}{\partial r} \right] + p \frac{\partial}{\partial y} \left[ \frac{D_{pp}}{p^3} \left( \frac{\partial g_e}{\partial y} - 4g_e \right) \right] + p \frac{\partial}{\partial y} \left( \frac{b}{p^2 g_e} \right)
\]

- Spatial diffusion = Fermi I
- Momentum diffusion = Fermi II
- Coulomb/ Synchrotron/iC losses

\[g_e = f_e \cdot p^4, \ y = \ln(p / m_e c)\]
The spherical shock slows down and its Mach number decreases in time.

Table 1. Parameters for Model Spherical Shocks

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_X$</th>
<th>$M_{\text{radio}}$</th>
<th>$M_{s,i}$</th>
<th>$kT_1$ (keV)</th>
<th>$B_1$ ($\mu G$)</th>
<th>$t_{\text{obs}}$ (Myr)</th>
<th>$M_{s,\text{obs}}$</th>
<th>$kT_{2,\text{obs}}$ (keV)</th>
<th>$u_{s,\text{obs}}$ (km s$^{-1}$)</th>
<th>$N$ ($10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sausage</td>
<td>2.7</td>
<td>4.6</td>
<td>4.0</td>
<td>2.1</td>
<td>1</td>
<td>211</td>
<td>3.21</td>
<td>8.6</td>
<td>$2.4 \times 10^3$</td>
<td>1.2</td>
</tr>
<tr>
<td>Toothbrush</td>
<td>1.5</td>
<td>2.8</td>
<td>3.6</td>
<td>3.0</td>
<td>1</td>
<td>144</td>
<td>3.03</td>
<td>11.2</td>
<td>$2.7 \times 10^3$</td>
<td>5.0</td>
</tr>
</tbody>
</table>

$M_X$: Mach number inferred from X-ray observations

$M_{\text{radio}}$: Mach number estimated from observed radio spectral index at the relic edge

$M_{s,i}$: initial shock Mach number at the onset of the simulations ($t_{\text{age}} = 0$)

$kT_1$: gas temperature in the preshock ICM

$B_1$: magnetic field strength in the preshock ICM

$t_{\text{obs}}$: shock age when the simulated results match the observations

$M_{s,\text{obs}}$: shock Mach number at $t_{\text{obs}}$

$kT_{2,\text{obs}}$: postshock temperature at $t_{\text{obs}}$

$u_{s,\text{obs}}$: shock speed at $t_{\text{obs}}$

$N = P_{\text{CR}e}/P_g$: the ratio of seed CR electron pressure to gas pressure in the preshock region

\[ D_{pp} \approx \frac{p^2}{4\tau_{\text{acc}}}, \quad \tau_{\text{acc}} \approx 10^8 \text{ yr} \]

The spherical shock slows down and its Mach number decreases in time.
Surface brightness profile

\[ I_\nu(R) = \int_0^{h_{1,\text{max}}} j_\nu(r) dh_1 + \int_0^{h_{2,\text{max}}} j_\nu(r) dh_2 \]

Radio flux density profile

\[ S_\nu(R) = \int_{\Omega_{\text{beam}}} I_\nu(R) d\Omega \quad \text{Smoothed over telescope beam} \]
Fitting of Radio Flux Profiles

Sausage Relic

- $M_{s,i} = 4.0$
- $\psi = 10^\circ$
- $153\,\text{MHz}$
- $608\,\text{MHz}$
- $L_{\text{cloud}} = 624\,\text{kpc}$
- $t_{\text{age}} = 211\,\text{Myr}$

Toothbrush Relic

- $M_{s,i} = 3.6$
- $\psi_1 = 12^\circ$
- $\psi_2 = 20^\circ$
- $153\,\text{MHz}$
- $608\,\text{MHz}$
- $L_{\text{cloud}} = 624\,\text{kpc}$
- $t_{\text{age}} = 144\,\text{Myr}$

Radiative cooling only

Turbulent acceleration with $\tau_{\text{acc}} = 10^8\,\text{yr}$
Fitting of Radio Integrated Spectra

Stroe et al. 2016 show reasonable agreement with observed data.

SZ corrected flux by Basu et al. 2016

black solid lines: at time of observations, $t_{\text{obs}}$

show reasonable agreement with observed data.
- Shock Acceleration model with $M \sim 3$ shock & postshock turbulence acceleration can reproduce observed profiles of radio flux $S_\nu(R)$ & integrated spectrum $J_\nu$ of the Sausage and the Toothbrush relic.

- need to understand better the properties of possible turbulence generated behind weak ICM shocks.

- need to study further collisionless shocks in $\beta=100$ plasma.

- These radio relics provide observational signatures of shocks in galaxy clusters.