Toward the Origin of Asteroid Geometries: Numerical Simulation of Planetesimal Collisions Using Smoothed Particle Elastic Dynamics

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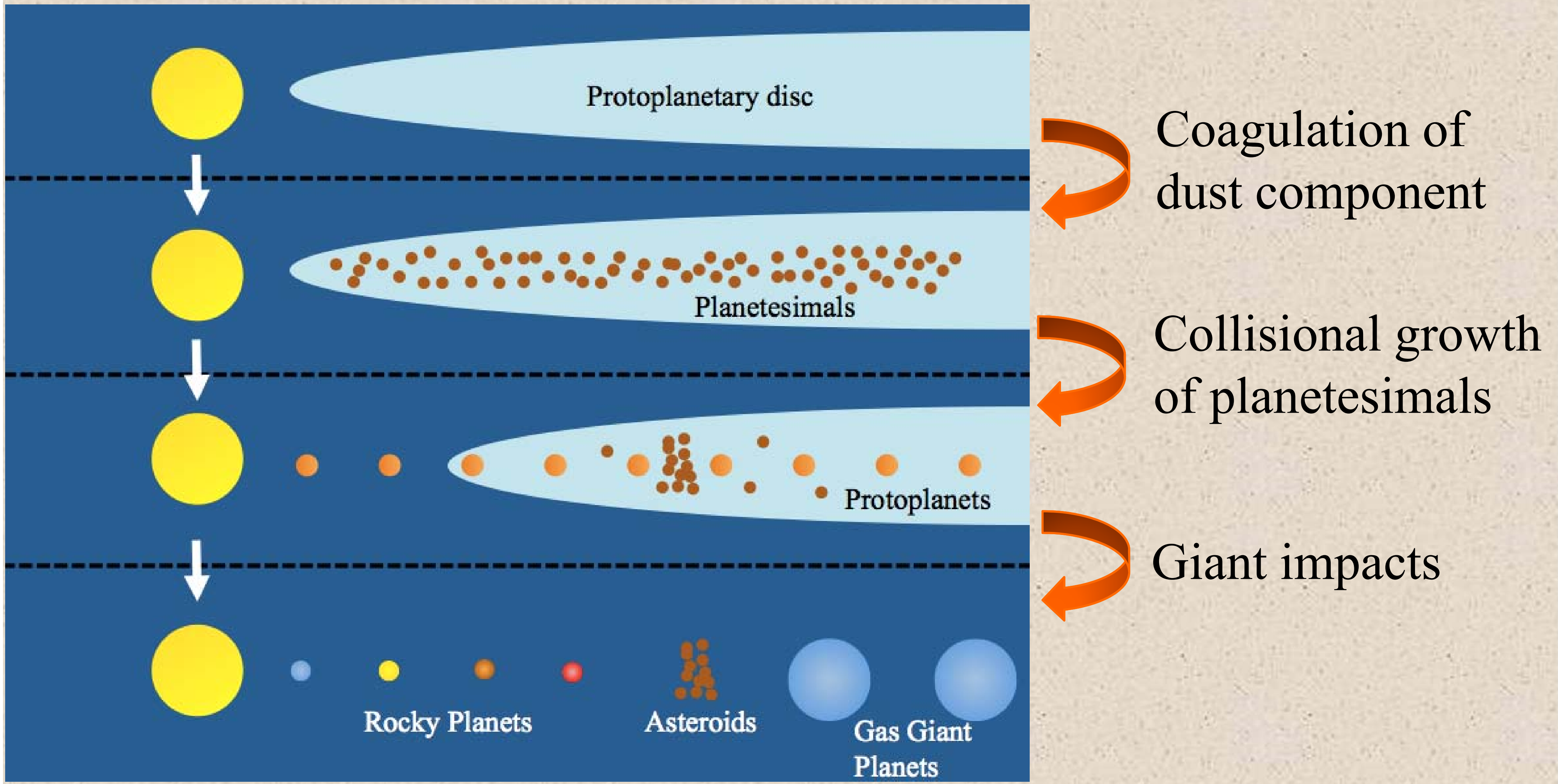
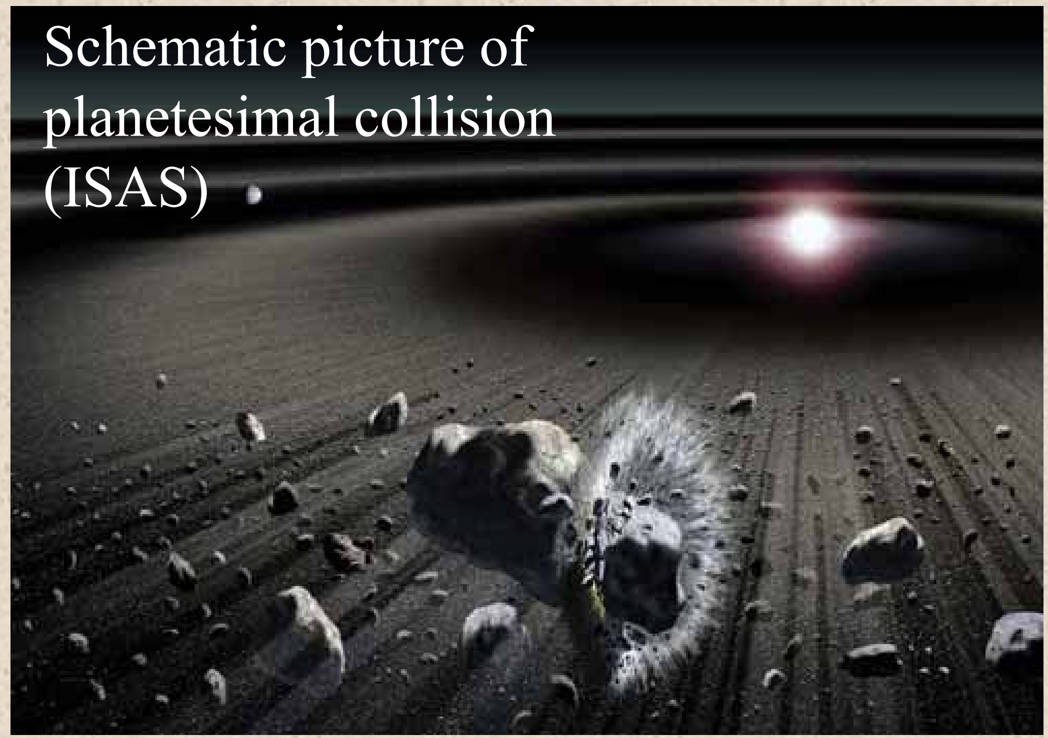
Introduction

Standard Solar System Formation Scenario



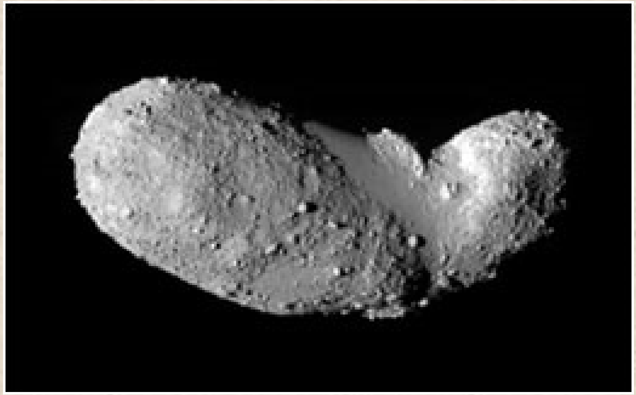
Planetary system is formed through planetesimal collisions.

Motivation



Planetesimal collision
=>Complex shape of asteroids

Asteroid Itokawa



Picture by
Spacecraft Hayabusa (JAXA)

Owing to detailed light curve observations,
we have shape models of about 1,000 asteroids (DAMIT database).

Clarifying the relationship between
asteroids' complicated shapes and impact conditions
=> Important clues to reveal the history of the solar system
(Orbits, number density, mean eccentricity etc.)

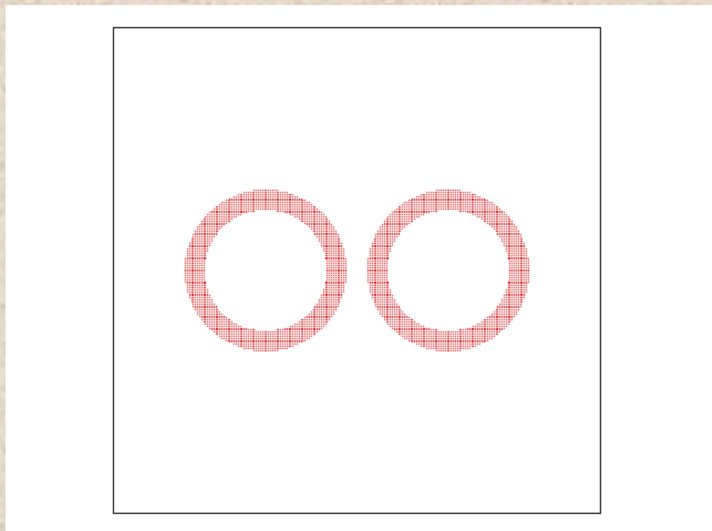
Purpose and method of this study

Purpose of this study

Clarifying the relationship between **impact conditions** and **shapes of planetesimals** due to destruction and reaccumulation

Method

- Smoothed Particle Elastic Dynamics (SPED) (Libersky and Petschek 1990)



rubber ring collision (Sugiura and Inutsuka 2017)

- Model for fracture of rocks (Benz and Asphaug 1995)

collisional destruction of planetesimal

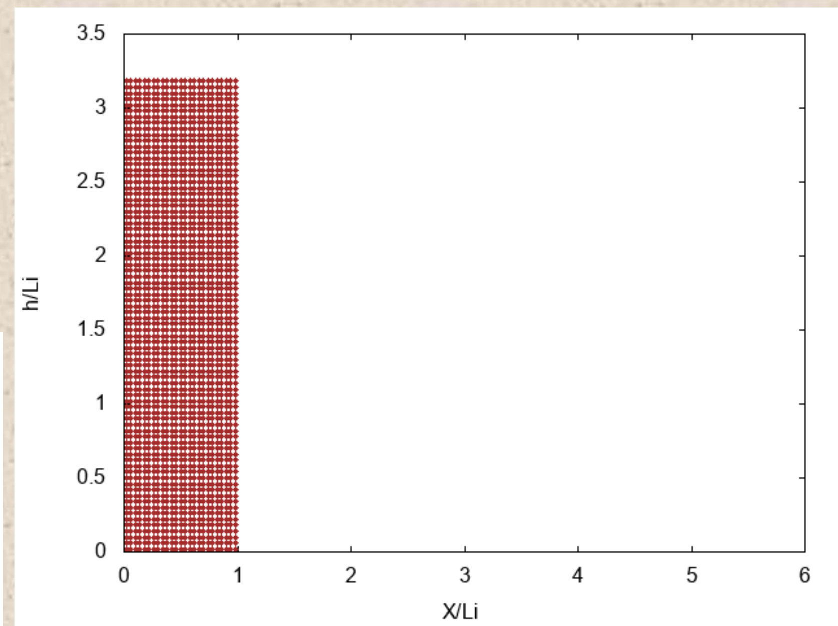
Purpose and method of this study

- Model for friction between destructed rock (granular material) (Jutzi 2015)

Comparison with the experiment of cliff collapse

time

Experiment (Lajeunesse et al. 2005)



our simulation

Combining **SPED** and **fracture/friction model**, we can reproduce shape formation due to

collisional destruction

and

reaccumulation of fragments.

Simulations of planetesimal collision

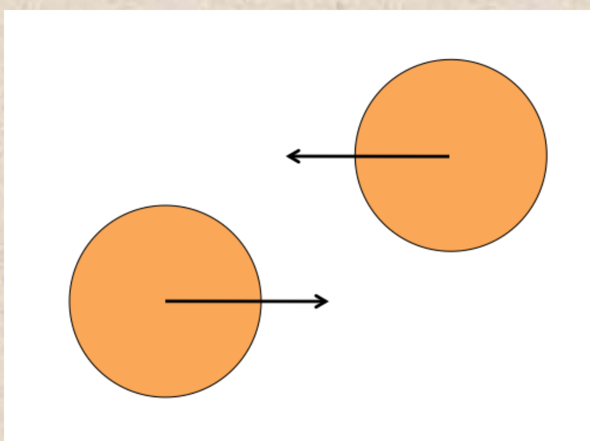
➢For simplicity, we consider
- **basaltic** plantesimals with **initial shape of sphere**,
- collisions between **equal size planetesimals** with radius of **50km**,
- friction coefficient of $\mu_d = \tan(40^\circ) = 0.839$.

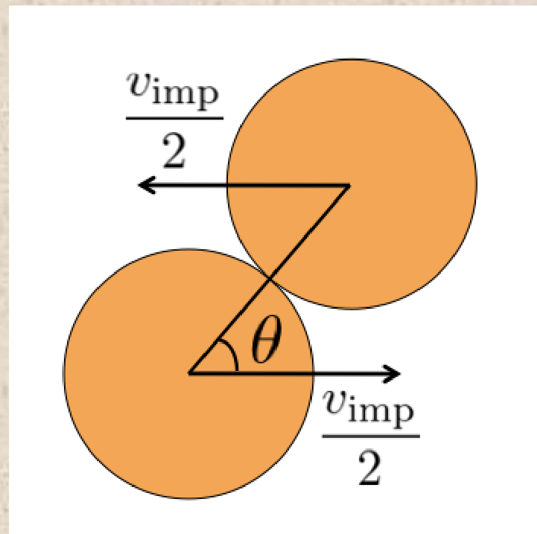
➢Resolution:
50,000 SPED particles for a sphere

➢We carried out simulations of collisions with the impact conditions of
- **velocity** from **50[m/s] to 400[m/s]**
- **angle** from **5[degree] to 45[degree]**

(angle of 0[degree] = head on collision)

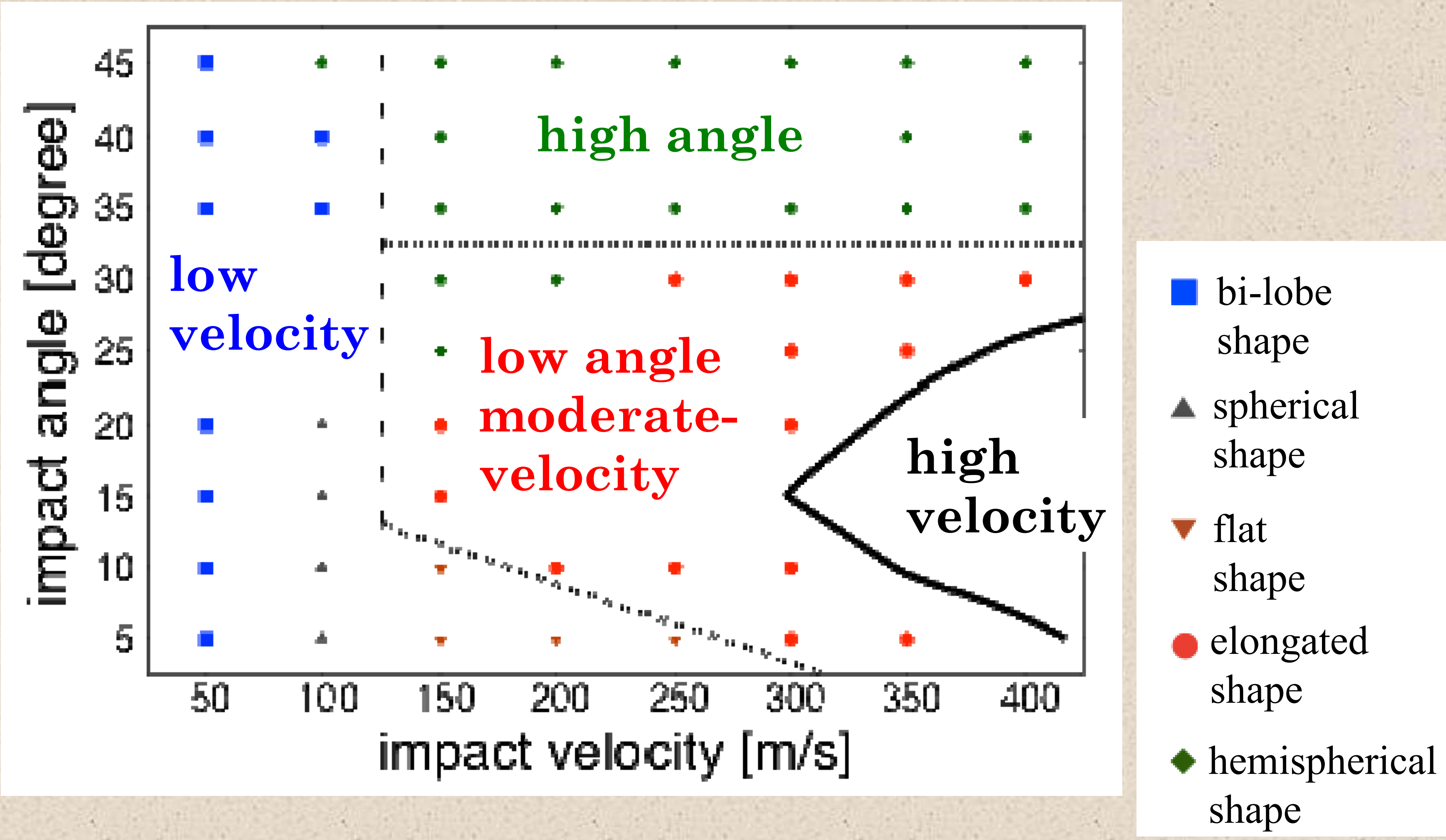
➢We measured **shapes of largest remnants** at the end of each impact simulation.



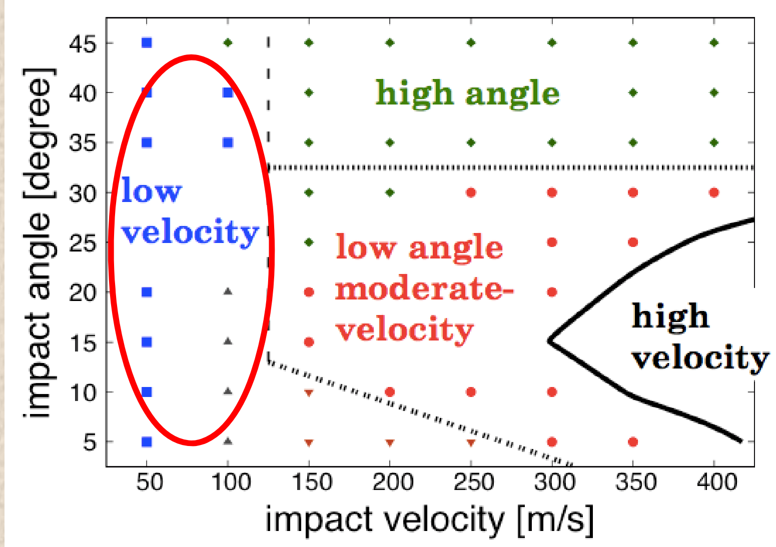


Result

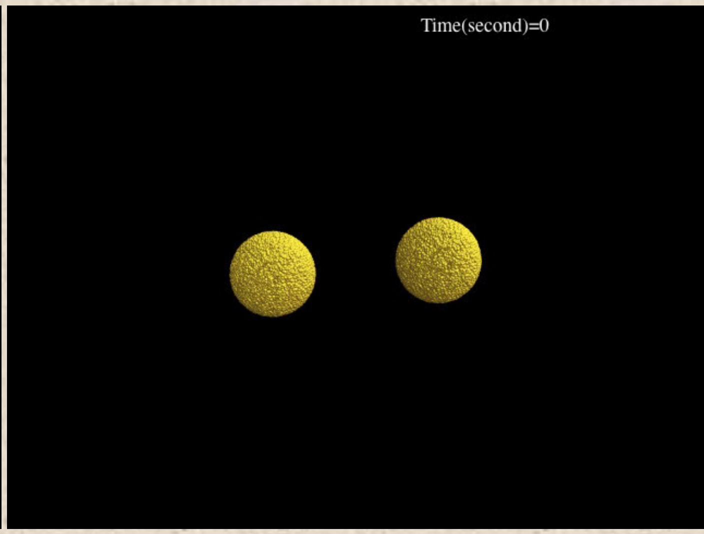
- ➢Type of geometry formed by collisional destruction and reaccumulation

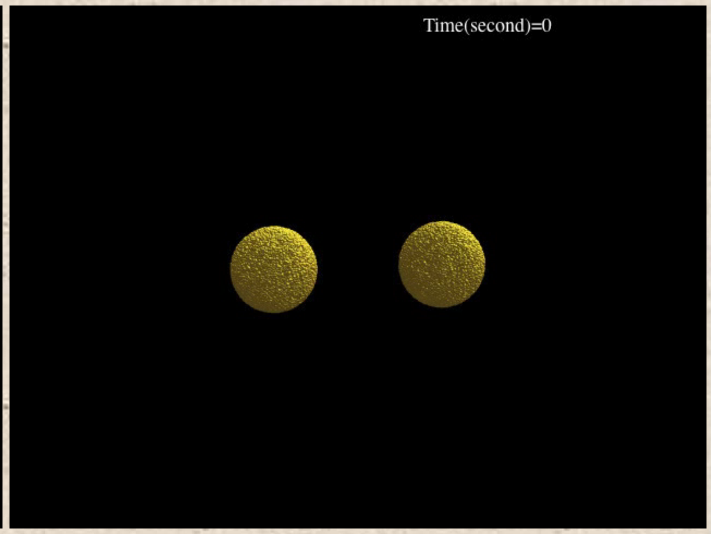


Result



●Low velocity
=> bi-lobe / spherical / flat shape





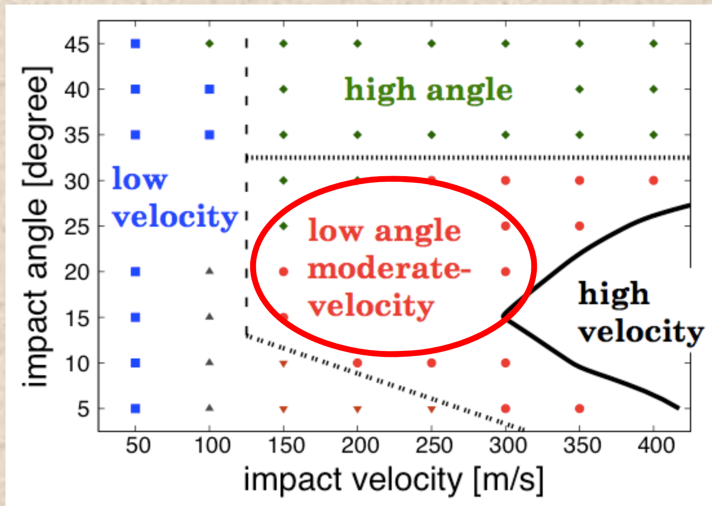
$\theta = 30^\circ$, $v_{\rm imp} = 50[{\rm m/s}]$
bi-lobe shape

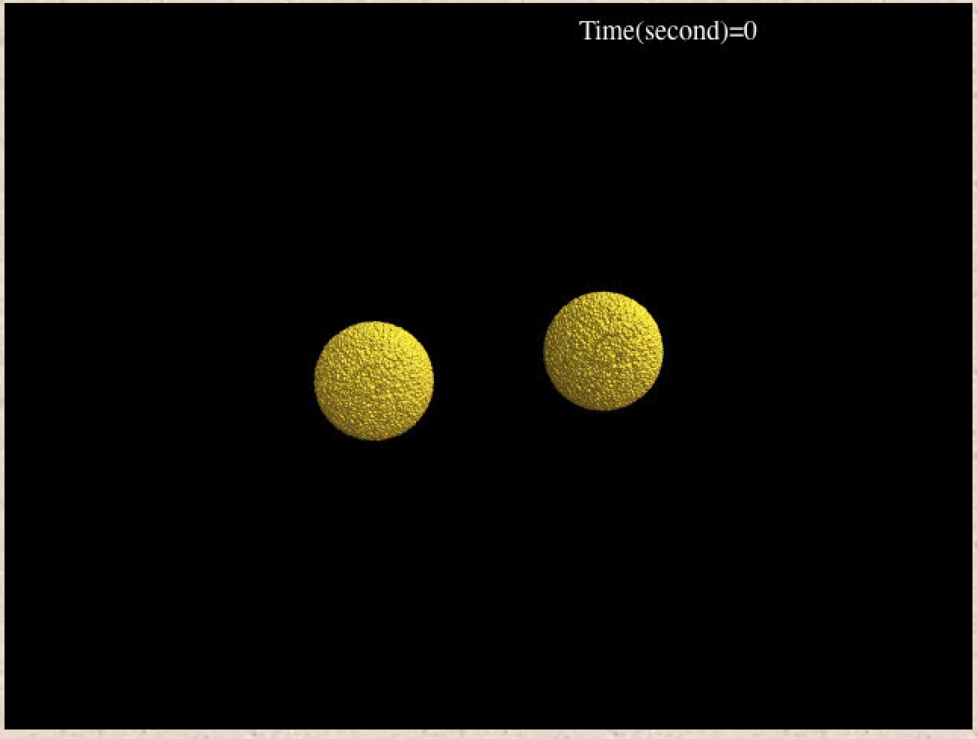
$\theta = 10^\circ$, $v_{\rm imp} = 100[{\rm m/s}]$
spherical shape

$\theta = 5^\circ$, $v_{\rm imp} = 200[{\rm m/s}]$
flat shape

Result

- **Low angle and moderate velocity => elongated shape**





$\theta = 15°$,

$v_{\mathrm{imp}} = 200[\mathrm{m/s}]$

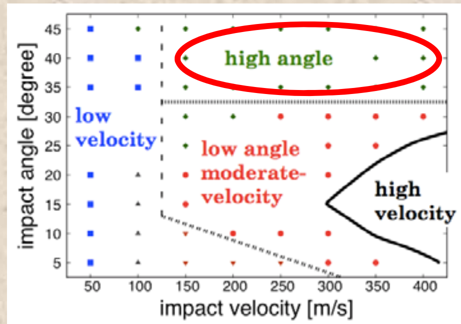
merging

$\theta = 20°$,

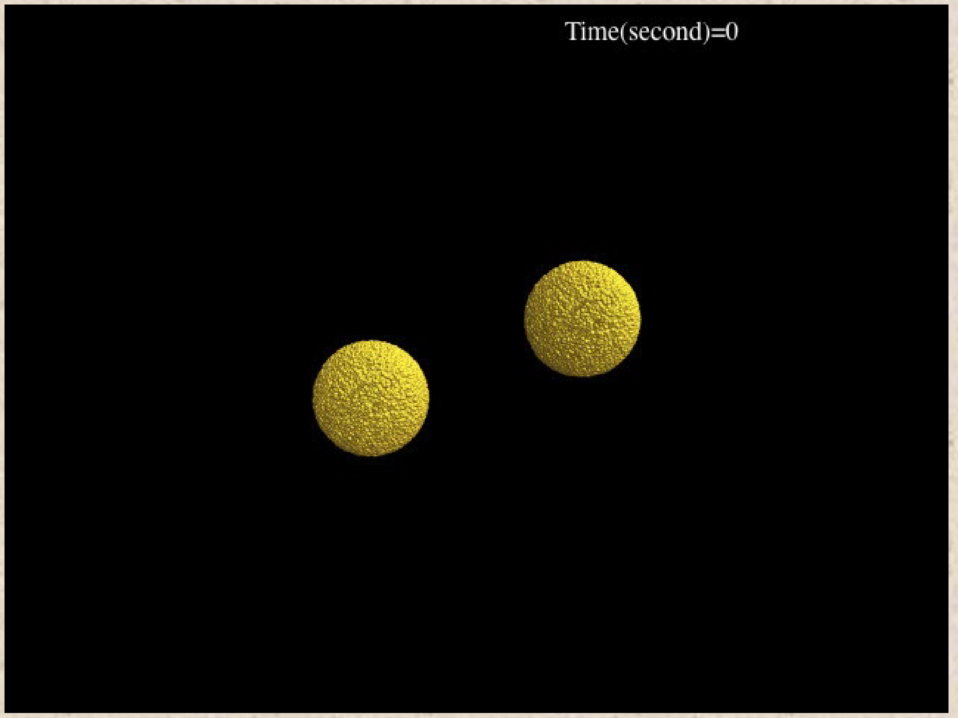
$v_{\mathrm{imp}} = 300[\mathrm{m/s}]$

non-merging

Result

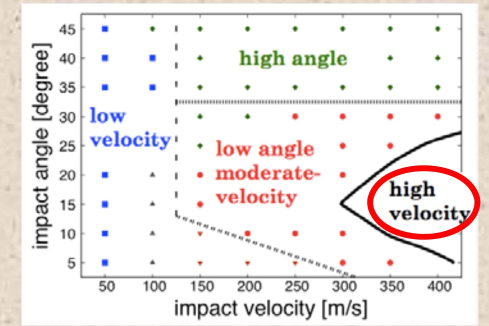


●High angle
=> hemispherical shape

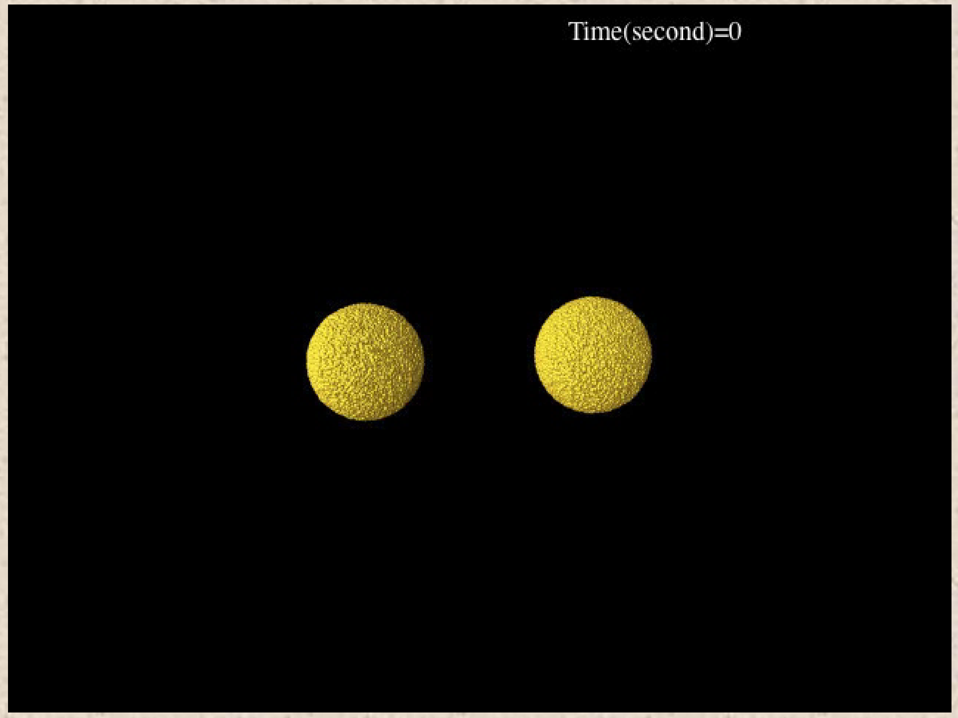


$\theta = 45°,$

$v_{\mathrm{imp}} = 350[\mathrm{m/s}]$



●High velocity
=> catastrophic destruction



$\theta = 5°,$

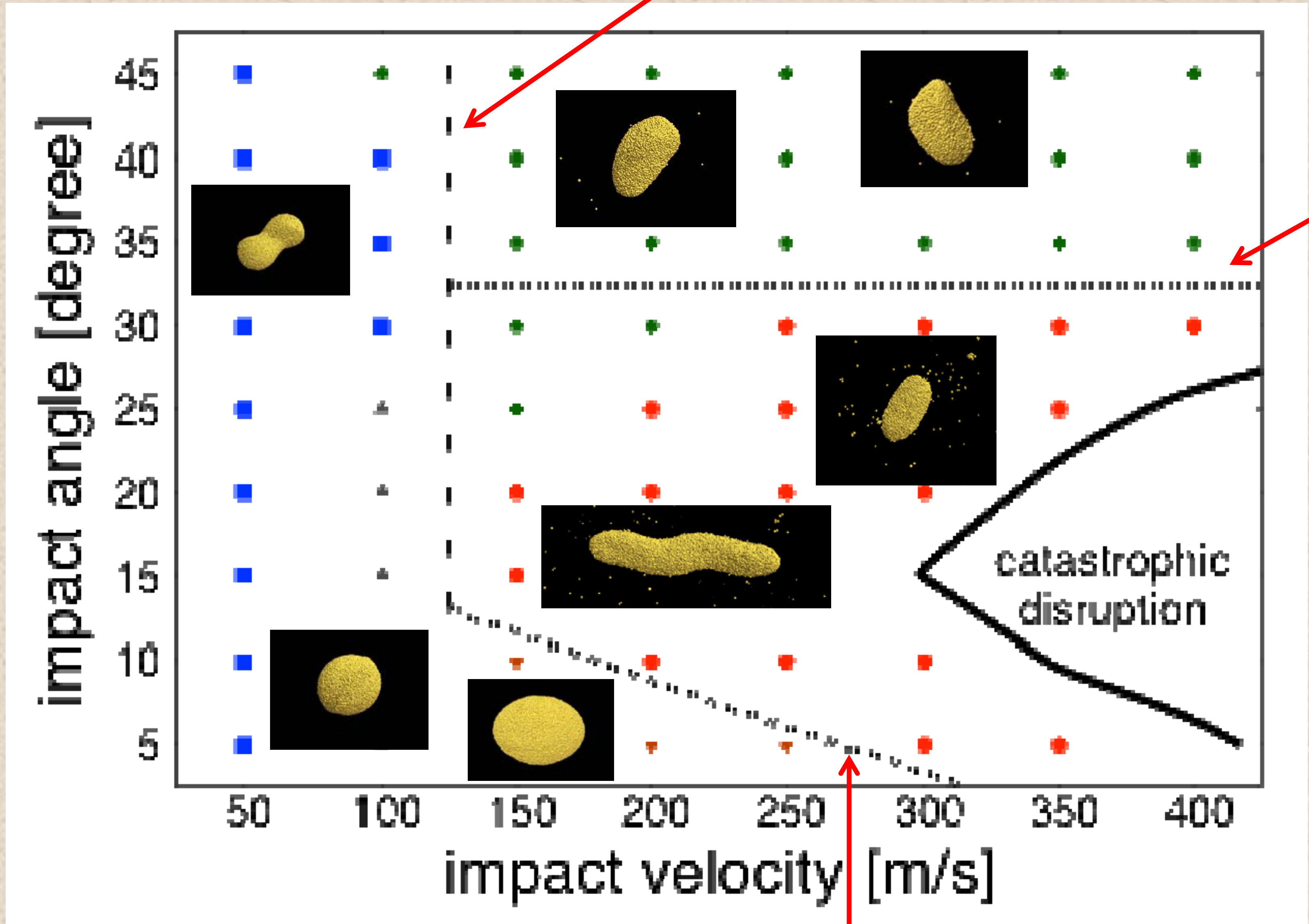
$v_{\mathrm{imp}} = 400[\mathrm{m/s}]$

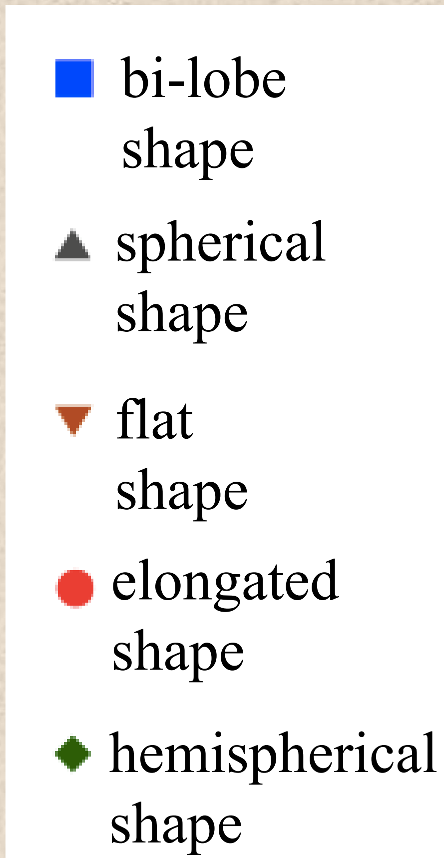
Result

➢Type of geometry formed by collisional destruction and reaccumulation

$v_{\mathrm{imp}} \sim 100[\mathrm{m/s}]$

$\theta \sim 30^{\circ}$

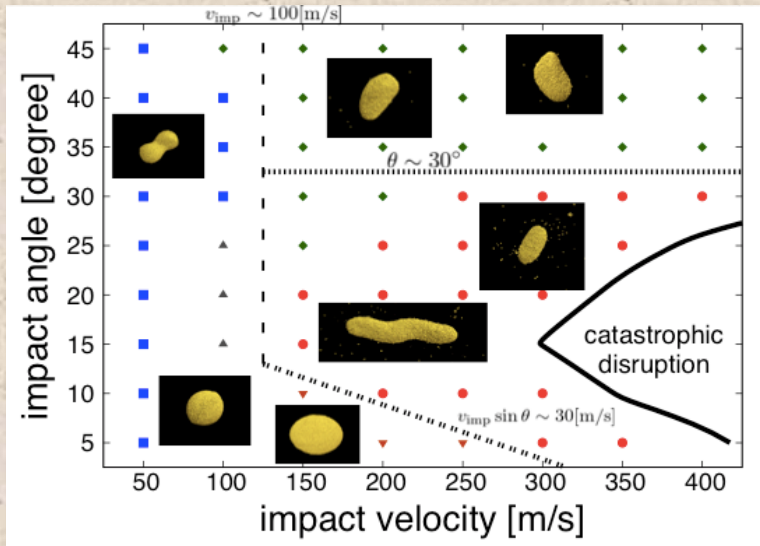




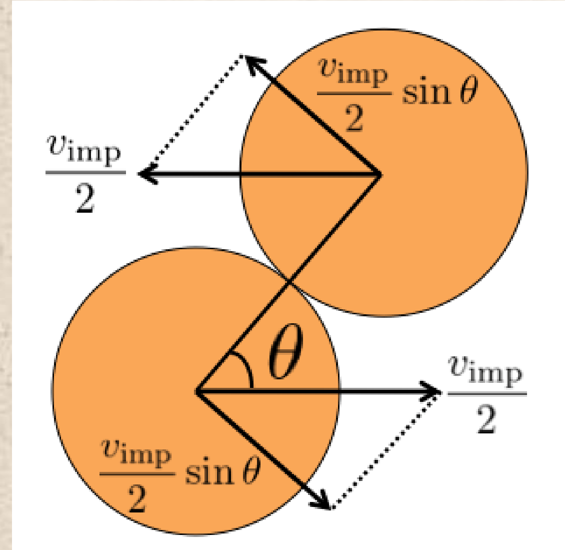
$v_{\mathrm{imp}} \sin\theta \sim 30[\mathrm{m/s}]$

Discussion

<u>What is the meaning of three lines distinguishing categories?</u>



➤ $v_{imp} \sim 100[m/s]$
critical dissipated energy by friction
= initial kinetic energy

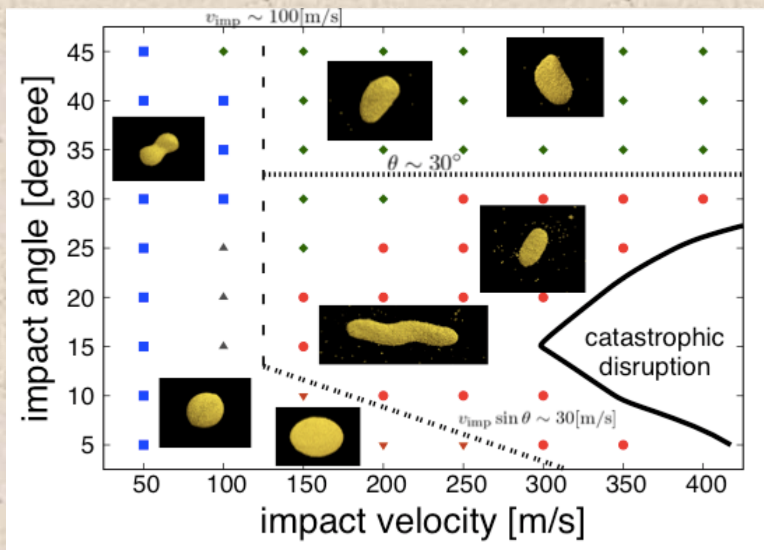


➤ $v_{imp} \sin\theta \sim 30[m/s]$
critical shear velocity
(cf. two body escape velocity ~ 60[m/s])

➤ $\theta \sim 30°$
critical impact angle
(half of target is directly affected by impactor)

For collisions between 50km planetesimals,
elongated shapes can be formed if
$v_{imp} > 100[m/s],\ v_{imp}\sin\theta > 30[m/s],\ \theta < 30°$

Summary and future work



Summary

- We performed **numerical simulations** for **planetesimal collisions** .
- We found that **irregular-shaped asteroids can be formed** through planetesimal collisions.
- For collisions between 50km planetesimals, we found that the **elongated shapes** can be formed if

$$v_{\rm imp} > 100[{\rm m/s}],\ \ v_{\rm imp}\sin\theta > 30[{\rm m/s}],\ \ \theta < 30°$$

Future work

- Collisions with different size planetesimals
- Collisions with large mass ratio

Analyze formation history of asteroids!

Thank you for your careful attention!

Elastic Dynamics

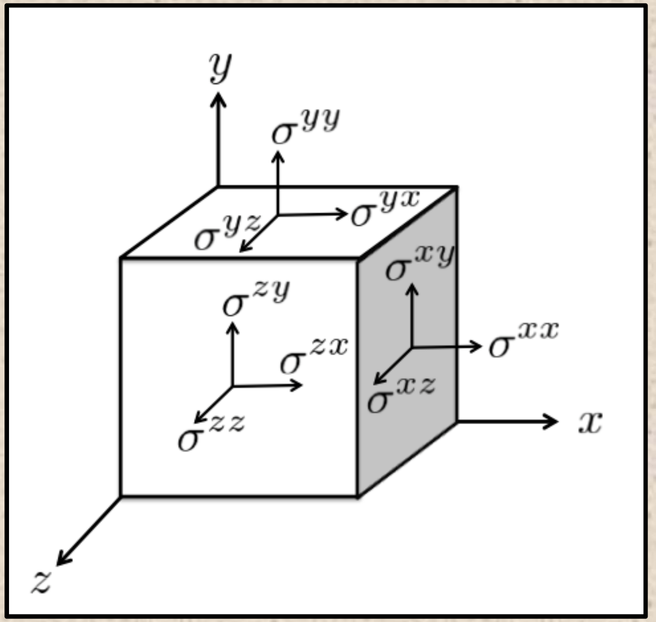
Equation of motion for elastic dynamics

$$\frac{dv^\alpha}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}$$

$\alpha, \beta = x, y, z$

Use summation rule

Lagrange time derivative



An element of elastic body

Here, $\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + S^{\alpha\beta}$

Stress tensor ($\sigma^{\alpha\beta}$)

Pressure (P) =isotropic part

Deviatoric stress tensor ($S^{\alpha\beta}$) =unisotropic part

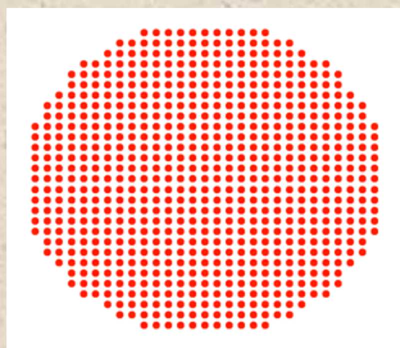
Pressure for elastic body, $P = C_s^2(\rho - \rho_0)$

Bulk sound speed (C_s)

Reference density (ρ_0)

Pressure can be negative for stretched (low density) part.

Smoothed Particle Hydrodynamics (SPH) method



Representation of sphere by SPH particles

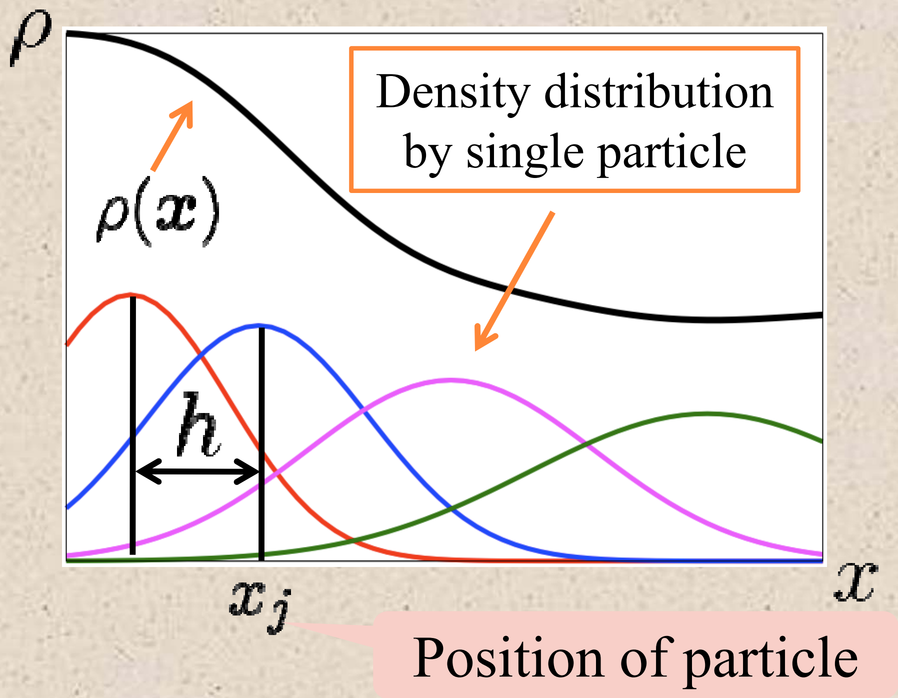
Density distribution

$$\rho(\boldsymbol{x}) = \sum_j m_j W(\boldsymbol{x} - \boldsymbol{x}_j, h)$$

j ← Particle index

h: Smoothing length
(Broadening length of particle)

$W(\boldsymbol{x}, h)$: Kernel function
(Broadening shape of particle)



SPH method is extended to elastic dynamics to simulate planetesimal collisions.
(e.g., Benz and Asphaug 1999, Jutzi 2015)

Standard SPH method

Approximation of standard SPH method

$$f_i = \sum_j m_j \frac{f_j}{\rho_j} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

gradient

$$\frac{\partial f_i}{\partial \boldsymbol{x}_i} = \sum_j m_j \frac{f_j}{\rho_j} \frac{\partial}{\partial \boldsymbol{x}_i} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

Gaussian kernel function

$$W(\boldsymbol{x}, h) = \left(\frac{1}{h\sqrt{\pi}}\right)^d \exp\left(-\frac{|\boldsymbol{x}|^2}{h^2}\right)$$

Equation of motion

$$\frac{d\boldsymbol{v}}{dt} = -\frac{\nabla P}{\rho} = -\nabla\left(\frac{P}{\rho}\right) - \frac{P}{\rho^2}\nabla\rho$$

$$\dot{\boldsymbol{v}}_i = -\sum_j m_j \left[\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right] \frac{\partial}{\partial \boldsymbol{x}_i} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

Equation of energy

$$\frac{du}{dt} = -\frac{P}{\rho}\nabla \cdot \boldsymbol{v} = -\frac{1}{2}\left[\nabla \cdot \left(\frac{P\boldsymbol{v}}{\rho}\right) - \boldsymbol{v} \cdot \nabla\left(\frac{P}{\rho}\right)\right] - \frac{P}{2\rho^2}\left[\nabla \cdot (\boldsymbol{v}\rho) - \boldsymbol{v} \cdot \nabla\rho\right]$$

$$\dot{u}_i = \frac{1}{2}\sum_j m_j \left[\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right](\boldsymbol{v}_i - \boldsymbol{v}_j) \frac{\partial}{\partial \boldsymbol{x}_i} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

Equations for Elastic Dynamics: Summary

$$\frac{dv^\alpha}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}$$: Equation of motion

$$\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + S^{\alpha\beta}$$

$$P = \frac{K}{\rho_0}(\rho - \rho_0)$$: Equation of state

$$\frac{d\rho}{dt} = -\rho\frac{\partial v^\alpha}{\partial x^\alpha}$$: Equation of continuity

$$\frac{dS^{\alpha\beta}}{dt} = 2\mu(\dot{\epsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\epsilon}^{\gamma\gamma}) + S^{\alpha\gamma}R^{\beta\gamma} + S^{\beta\gamma}R^{\alpha\gamma}$$

:time evolution equation of deviatoric stress tensor

$$\dot{\epsilon}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha}\right) \quad R^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right)$$

$$\frac{dE}{dt} = \frac{1}{\rho}\sigma^{\alpha\beta}\frac{\partial v^\alpha}{\partial x^\beta}$$: Equation of energy

SPH Method

$$\frac{dv^\alpha}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}$$

$$\frac{\partial f_i}{\partial x_i^\alpha} = \sum_j m_j \frac{f_j}{\rho_j}\frac{\partial}{\partial x_i^\alpha} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

$$\Rightarrow \quad \frac{dv^\alpha}{dt} = \sum_j m_j \left[\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right]\frac{\partial}{\partial x_i^\beta} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

For spatial derivative of velocity,

$$\frac{\partial v_i^\beta}{\partial x_i^\alpha} = \sum_j m_j \frac{v_j^\beta - v_i^\beta}{\rho_j}\frac{\partial}{\partial x_i^\alpha} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

$$\frac{d\rho}{dt} = -\rho\frac{\partial v^\alpha}{\partial x^\alpha}$$

$$\Rightarrow \quad \frac{d\rho}{dt} = -\rho_i \sum_j m_j \frac{v_j^\alpha - v_i^\alpha}{\rho_j}\frac{\partial}{\partial x_i^\alpha} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

$$\frac{dE}{dt} = \frac{1}{\rho}\sigma^{\alpha\beta}\frac{\partial v^\alpha}{\partial x^\beta}$$

$$\Rightarrow \quad \frac{dE}{dt} = \frac{\sigma_i^{\alpha\beta}}{\rho_i}\sum_j m_j \frac{v_j^\alpha - v_i^\alpha}{\rho_j}\frac{\partial}{\partial x_i^\beta} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

SPH Method

$$\frac{\partial v_i^\beta}{\partial x_i^\alpha} = \sum_j m_j \frac{v_j^\beta - v_i^\beta}{\rho_j} \frac{\partial}{\partial x_i^\alpha} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

$$\frac{dS^{\alpha\beta}}{dt} = 2\mu(\dot{\epsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\epsilon}^{\gamma\gamma}) + S^{\alpha\gamma}R^{\beta\gamma} + S^{\beta\gamma}R^{\alpha\gamma}$$

$$\dot{\epsilon}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha}\right) \quad R^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right)$$

$$\Rightarrow \quad \frac{dS^{\alpha\beta}}{dt} = 2\mu(\dot{\epsilon}_i^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\epsilon}_i^{\gamma\gamma}) + S_i^{\alpha\gamma}R_i^{\beta\gamma} + S_i^{\beta\gamma}R_i^{\alpha\gamma}$$

$$\dot{\epsilon}_i^{\alpha\beta} = \frac{1}{2}\sum_j m_j \left[\frac{v_j^\alpha - v_i^\alpha}{\rho_j}\frac{\partial}{\partial x_i^\beta} + \frac{v_j^\beta - v_i^\beta}{\rho_j}\frac{\partial}{\partial x_i^\alpha}\right] W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

$$R_i^{\alpha\beta} = \frac{1}{2}\sum_j m_j \left[\frac{v_j^\alpha - v_i^\alpha}{\rho_j}\frac{\partial}{\partial x_i^\beta} - \frac{v_j^\beta - v_i^\beta}{\rho_j}\frac{\partial}{\partial x_i^\alpha}\right] W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$$

We will follow the time evolution of $\boldsymbol{x}, \ \boldsymbol{v}, \ \rho, \ E, \ S^{\alpha\beta}$

Tillotson equation of state

EoS of gas + elastic body

Parameters are derived from laboratory experiment.

- high density or low density and low temperature

$$P = \left[a + \frac{b}{(u/u_0\eta^2) + 1}\right]\rho u + A\mu + B\mu^2$$

gas part

elastic part

where $\eta = \rho/\rho_0$, $\mu = \eta - 1$

- low density and high temperature (gas like behavior)

$$P = a\rho u + \left[\frac{b\rho u}{(u/u_0\eta^2) + 1} + A\mu e^{-\beta(\rho_0/\rho - 1)}\right] e^{-\alpha(\rho_0/\rho - 1)^2}$$

Tillotson parameters

TABLE II

Tillotson Eos Parameters

	ρ_0 (g/cc)	A (erg/cc)	B (erg/cc)	E_0 (erg/g)	E_{iv} (erg/g)	E_{cv} (erg/g)	a	b	α	β
Basalt	2.7	$2.67\ 10^{11}$	$2.67\ 10^{11}$	$4.87\ 10^{12}$	$4.72\ 10^{10}$	$1.82\ 10^{11}$	0.5	1.50	5.0	5.0[a]
Ice	0.917	$9.47\ 10^{10}$	$9.47\ 10^{10}$	$1.00\ 10^{11}$	$7.73\ 10^{9}$	$3.04\ 10^{10}$	0.3	0.1	10.0	5.0[b]

(Benz and Asphaug 1999)

Elastic Dynamics

Stress tensor: internal force to reduce strain
If stress tensor is assumed to be proportional to strain (Hooke's law),

$$\sigma^{\alpha\beta} = 2\mu(\epsilon^{\alpha\beta} - \frac{1}{3}\epsilon^{\gamma\gamma}\delta^{\alpha\beta}) + K\epsilon^{\gamma\gamma}\delta^{\alpha\beta}$$

non-diagonal part diagonal part

μ : shear modulus K: bulk modulus $\epsilon^{\alpha\beta}$: strain tensor

$$\epsilon^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u^\alpha}{\partial x^\beta} + \frac{\partial u^\beta}{\partial x^\alpha}\right)$$

$\boldsymbol{u}$: displacement vector $\boldsymbol{u} \equiv \boldsymbol{x}(t) - \boldsymbol{x}(t=0)$

We define diagonal part as pressure: $P \equiv -K\epsilon^{\gamma\gamma}$
We define non-diagonal part as deviatoric stress tensor $S^{\alpha\beta}$

$$\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + S^{\alpha\beta} \quad S^{\alpha\beta} = 2\mu\left(\epsilon^{\alpha\beta} - \frac{1}{3}\epsilon^{\gamma\gamma}\delta^{\alpha\beta}\right)$$

Elastic Dynamics

$$P \equiv -K\epsilon^{\gamma\gamma}$$

Bulk modulus is written as $K = C_s^2 \rho_0$

Trace of strain tensor shows the change of volume of element

$$V' = (1 + \epsilon^{\gamma\gamma})V$$

V': specific volume after deformation

V: specific volume before deformation

$$\begin{cases} V = 1/\rho_0 \\ V' = 1/\rho \end{cases}$$

$$\Rightarrow \quad \epsilon^{\gamma\gamma} = \frac{\rho_0 - \rho}{\rho_0}$$

Therefore,

$$P = C_s^2(\rho - \rho_0)$$

Fracture Model

➢Fracture model (Benz and Asphaug 1995)

- We define damage parameter "*D*" for each SPH particle
  D=1 : disrupted SPH particle
  D=0 : intact SPH particle

- *D* can accumulate with local tensile loading if strain exceeds threshold. This threshold is determined by Weibull parameters.(Weibull 1965)

- *D*=1 particle cannot transmit tensile or shear force.

$$P_i \rightarrow (1 - D)P_i \text{ if } P_i < 0$$

$$S_i^{\alpha\beta} \rightarrow (1 - D)S_i^{\alpha\beta}$$

Friction Model

➢Friction model (Jutzi 2014)

To represent the friction between disrupted rocks, we modify the deviatoric stress tensor as follows:

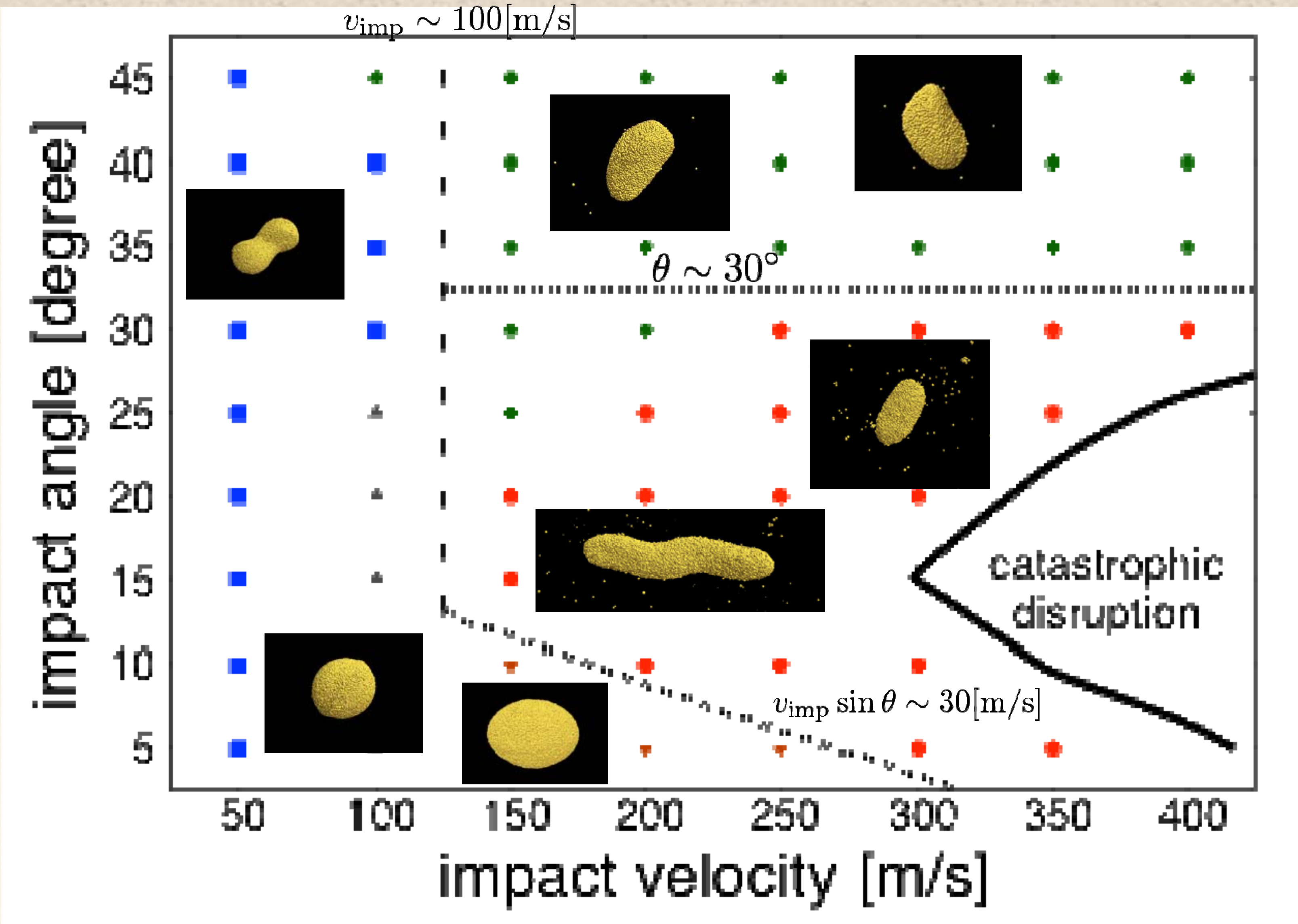
$$S_i^{\alpha\beta} \rightarrow f S_i^{\alpha\beta}$$

$$f = \min[Y_d/\sqrt{J_2}, 1], \;\; J_2 = \frac{1}{2} S^{\alpha\beta} S^{\alpha\beta}$$

$$Y_d = \underline{\mu_d} P$$

friction coefficient

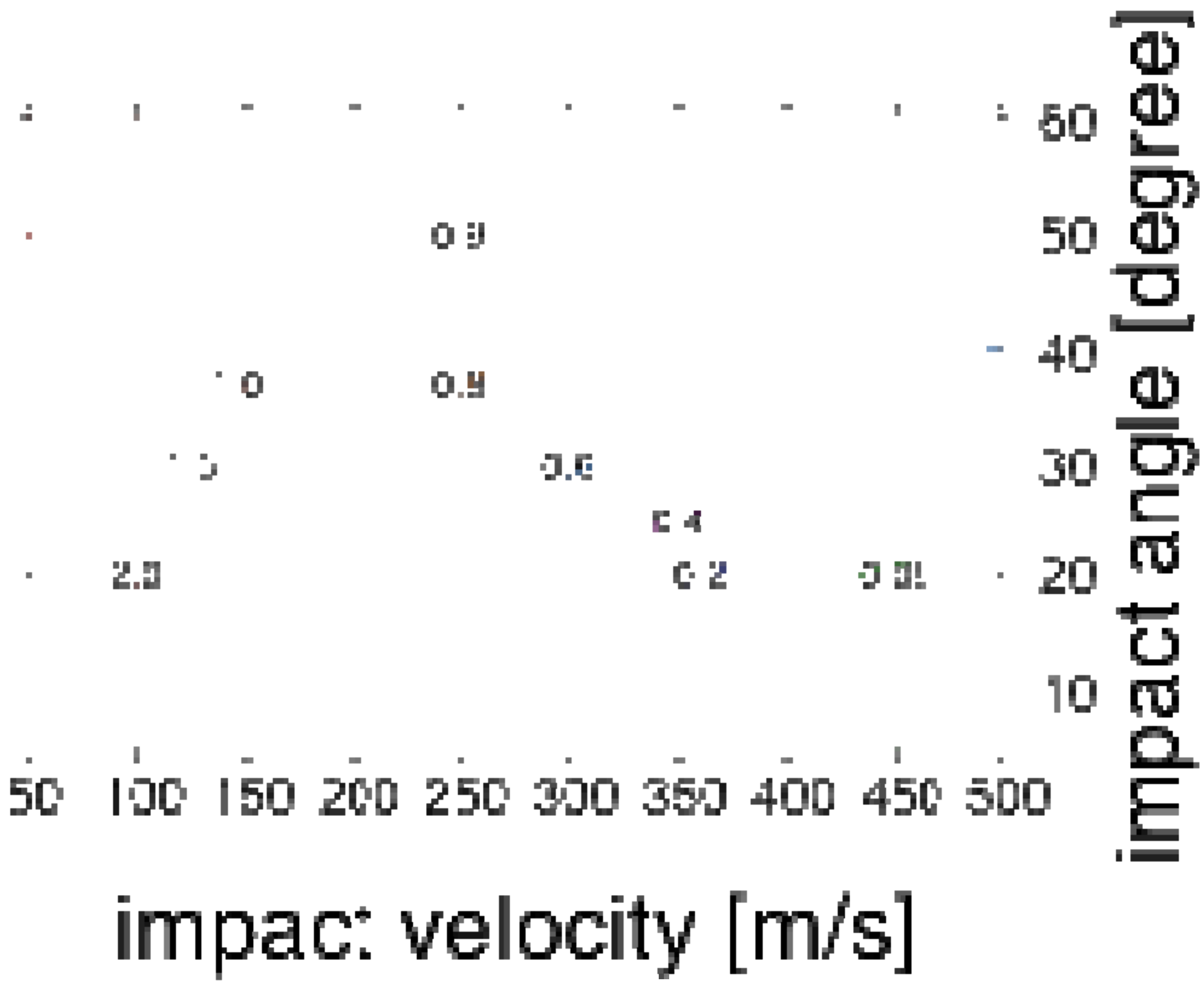
Result



$v_{imp} \sim 100[m/s]$
$\theta \sim 30°$
$v_{imp} \sin\theta \sim 30[m/s]$
catastrophic disruption
impact angle [degree]
impact velocity [m/s]
45
40
35
30
25
20
15
10
5
50
100
150
200
250
300
350
400

Result

Contour of $M_{\mathbf{largest\ remnant}}/M_{\mathbf{target}}$

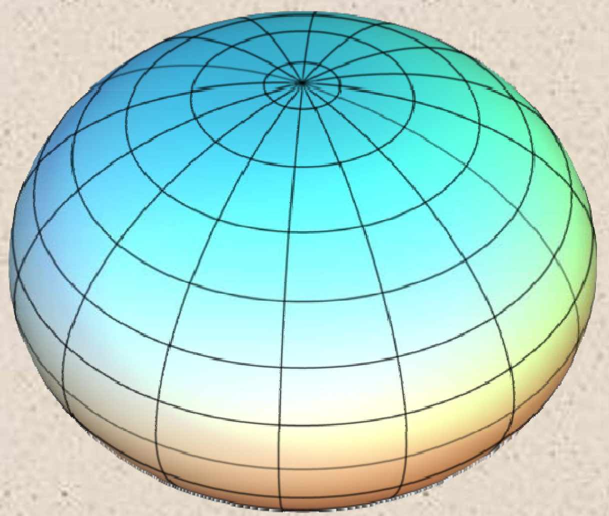


Measure the Shape: Axis Ratio

- Axis ratio is a good measure of the shape of body
- The way to determine axis ratio is following Fujiwara (1978)

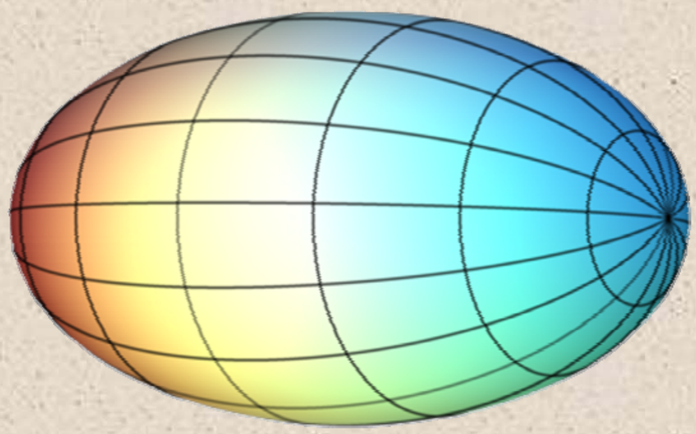
•length of minor axis **c**: the smallest space of two parallel plate when we put the object between two plates.
•length of major axis **a**: the largest space of two parallel plate for the direction perpendicular to c axis.
•length of intermediate axis **b**: space of two parallel plate for the direction perpendicular to both a and c axis

Oblate shape



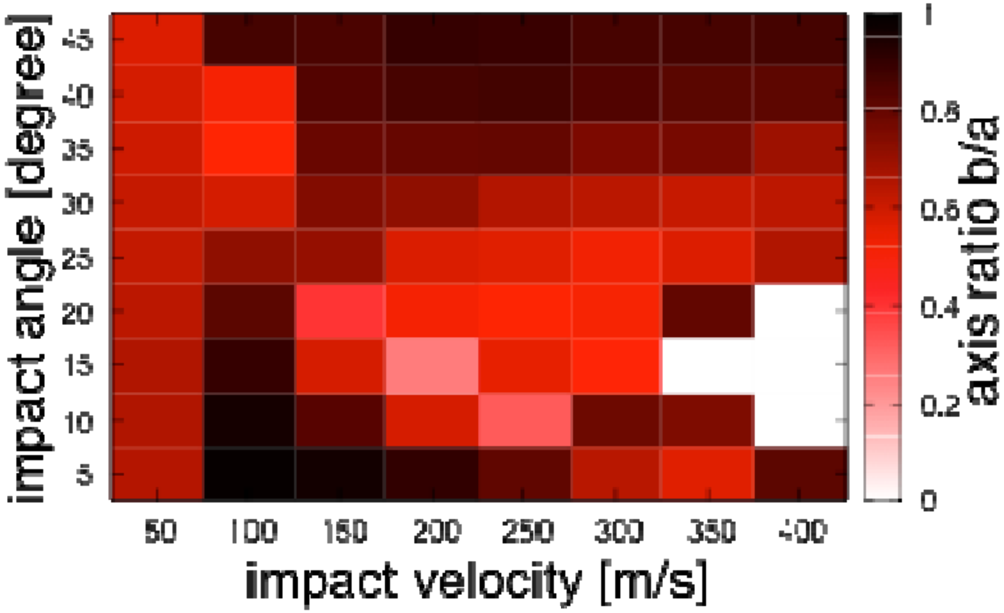
b/a~1 、

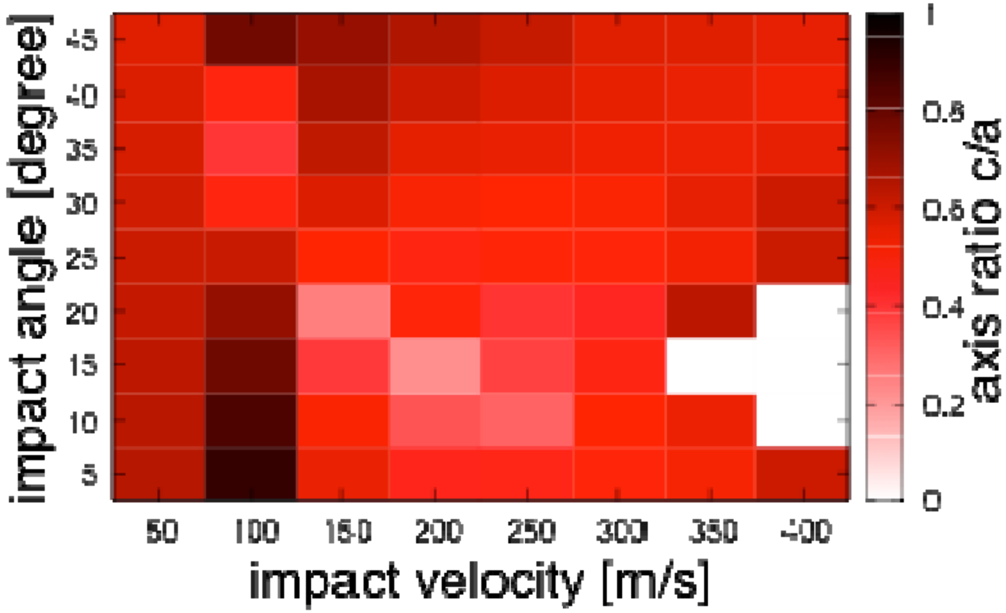
Prolate shape



b/a<1 、

Result





ratio of
intermediate axis length
and
major axis length

ratio of
minor axis length
and
major axis length

Critical dissipated energy

Estimate dissipated energy required to deform two granular sphere into one huge granular sphere in the existence of friction.

Dissipated energy due to friction per volume ~ $\mu_d P$

Typical pressure ~ central pressure due to the self-gravity

$$P \sim \frac{2}{3}\pi G\rho(\sqrt[3]{2})^2$$

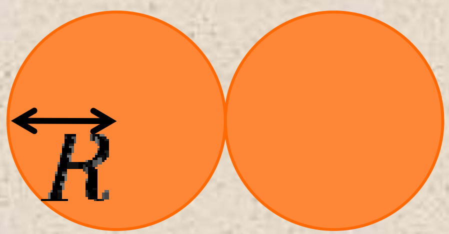
Required energy to deform whole volume of two sphere

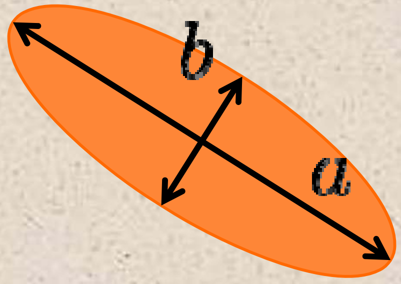
$$E_{\mathrm{dis}} \sim \mu P \frac{8}{3}\pi R^3$$

$$\frac{1}{2}M_t\left(\frac{v_{\mathrm{imp,crit}}}{2}\right)^2 \cdot 2 = E_{\mathrm{dis}} \quad \Rightarrow \quad v_{\mathrm{imp,crit}} = 61.00[\mathrm{m/s}]$$

Critical shear velocity

Estimate required gravitational potential change due to elongation





ellipsoid with $a > b = c$

$V_{\text{tot,ini}}$ $V_{\text{tot,el}}$

$$V_{\text{tot,ini}} = -\frac{3}{5}\frac{GM_t^2}{R_t} - \frac{1}{2}\frac{GM_t^2}{R_t} = -4.534 \times 10^{28}[\text{erg}]$$

If we assume $b/a = 0.3$

$$V_{\text{tot,el}} = -4.167 \times 10^{28}[\text{erg}]$$

$$\frac{1}{2}M_t\left(\frac{(v\sin\theta)_{\text{crit}}}{2}\right)^2 \cdot 2 = V_{\text{tot,ini}} - V_{\text{tot,el}} \Rightarrow (v\sin\theta)_{\text{crit}} = 32.20[\text{m/s}]$$

Note: energy dissipated by friction can be compensated by tangential velocity.