

# Suppression of extreme orbital evolution in triple systems with short-range forces

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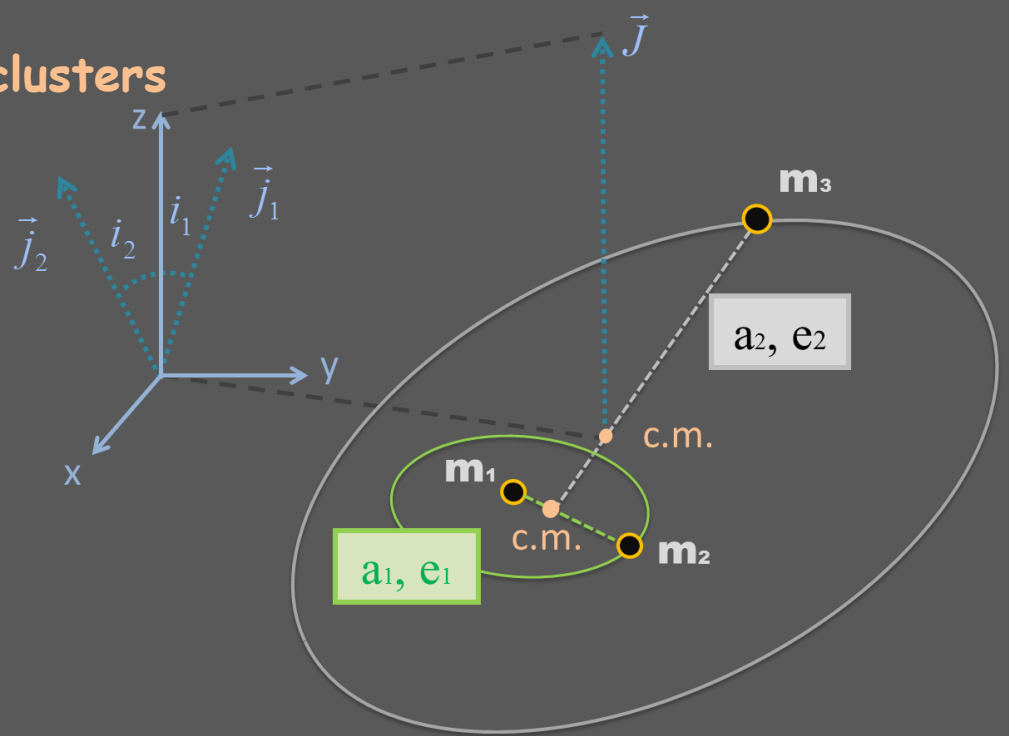
# Triple systems

Three-body systems are ubiquitous in astrophysics ~25%

- Planet-satellite systems
- Exoplanet systems (Hot Jupiters)
- Black holes in dense stellar clusters

...  
Hierarchical triple

- Inner binary
- Outer binary
- Stable



# Lidov-Kozai oscillations

## Hamiltonians

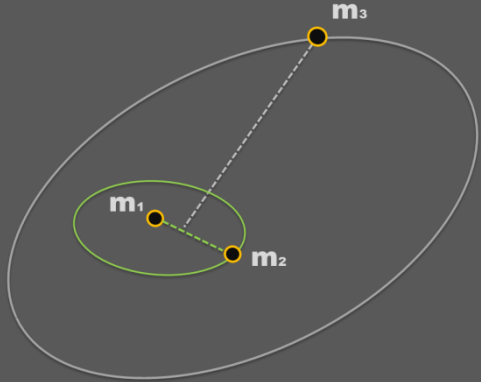
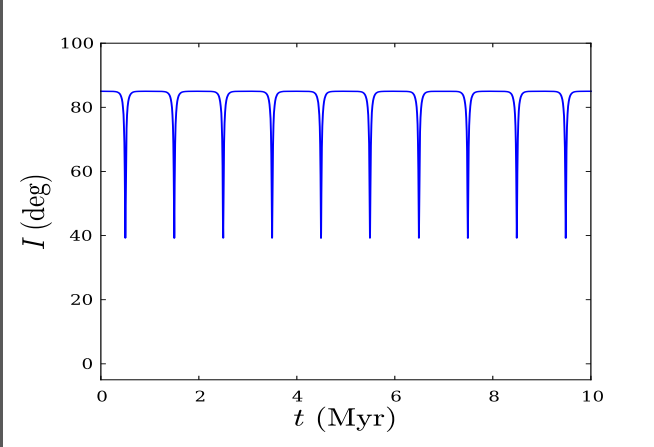
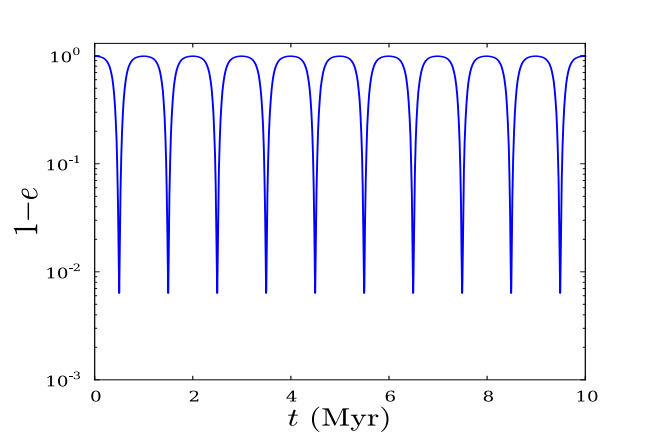
$$\begin{cases}
 H = -\frac{Gm_1m_2}{2a_1} - \frac{Gm_3(m_1+m_2)}{2a_2} + \Phi \\
 \Phi = -\frac{G}{a_2} \sum_{l=2}^{\infty} \left(\frac{a_1}{a_2}\right)^l m_1m_2m_3 \frac{m_1^{l-1} - (-m_2)^{l-1}}{(m_1+m_2)^l} \left(\frac{r_1}{a_1}\right)^l \left(\frac{a_2}{r_2}\right)^{l+1} P_l(\cos\theta)
 \end{cases}$$

## Theory

---Eccentricity and inclination oscillations induced if  $i > 40^\circ$

---If  $i$  large (85-90 degrees), get extremely large eccentricities ( $e > 0.99$ )

## Orbital (secular) evolution



Introduction

Oct (test mass)

Summary



SRFs

Oct (comparable mass)

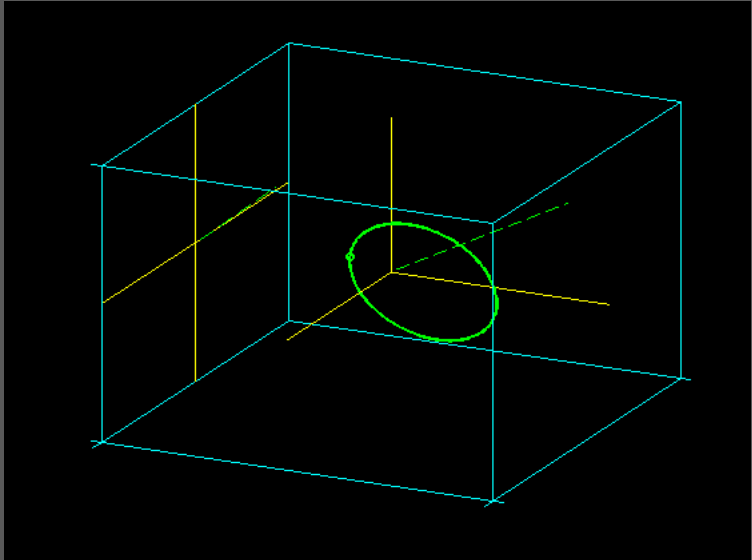
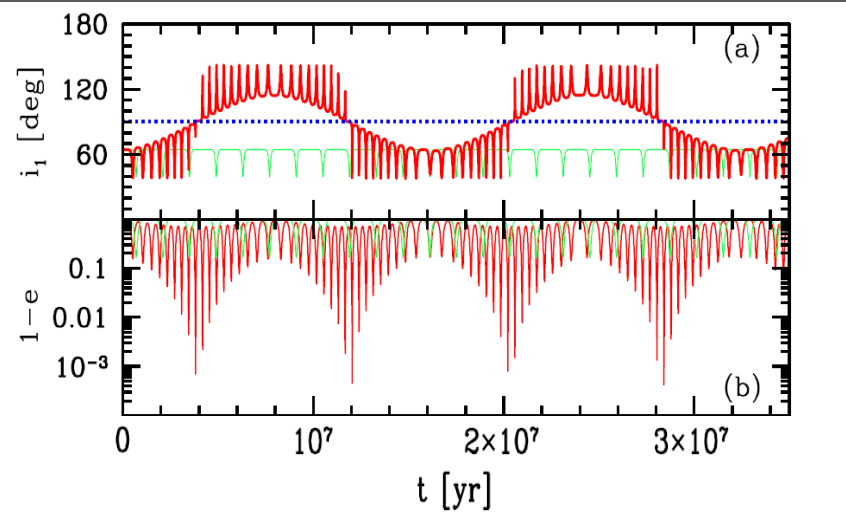
# Lidov-Kozai oscillations (Octupole)

## Orbital evolution

e.g. Ford et al 2000; Naoz et al 11; Katz et al 11;  
Lithwick & Naoz 11; Li et al 14; Petrovich 15; Antognini 15

Edit: Naoz

Naoz et al 11



## Octupole effect

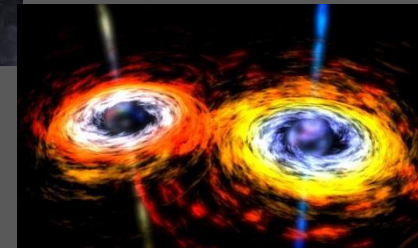
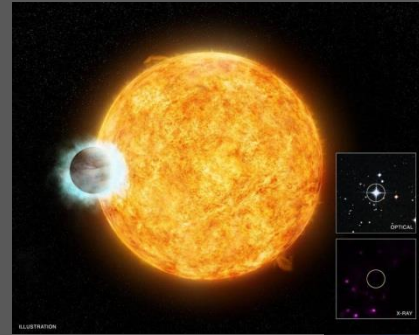
- Very large  $e$  even for modest  $i$
- $L_p$  can flip with respect to the outer binary





# Applications

- The excitation of eccentricities of exoplanet systems and the formation of HJ through high-eccentricity migration
- The production of Type Ia supernovae from white-dwarf binary mergers or direct collisions
- The formation and merger of stellar black hole binaries at the centers globular clusters or galaxies
- The formation and merger of supermassive black hole binaries



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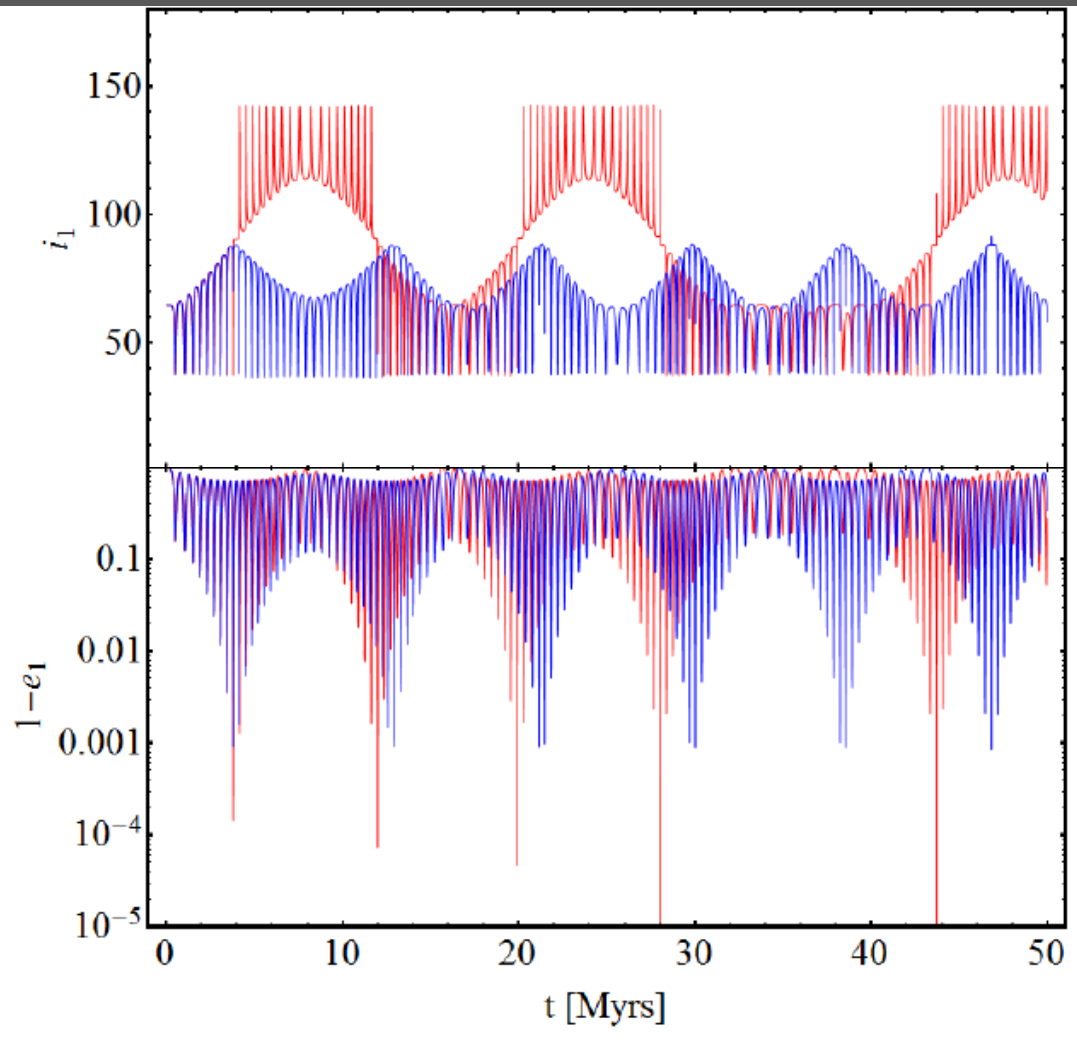


SRFs

Oct (comparable mass)

# Short-Range Forces

$m_0=1M_{\text{sun}}, m_1=1M_J, m_3=0.04M_{\text{sun}}, e_1=0.001, e_2=0.6$   
 $a_1=6 \text{ AU}, a_2=100 \text{ AU}$



## Includes:

- (1) precession of periapse (GR)
- (2) tidal bulge of the planet induced by the star
- (3) planet oblateness (rotation)

## Effects:

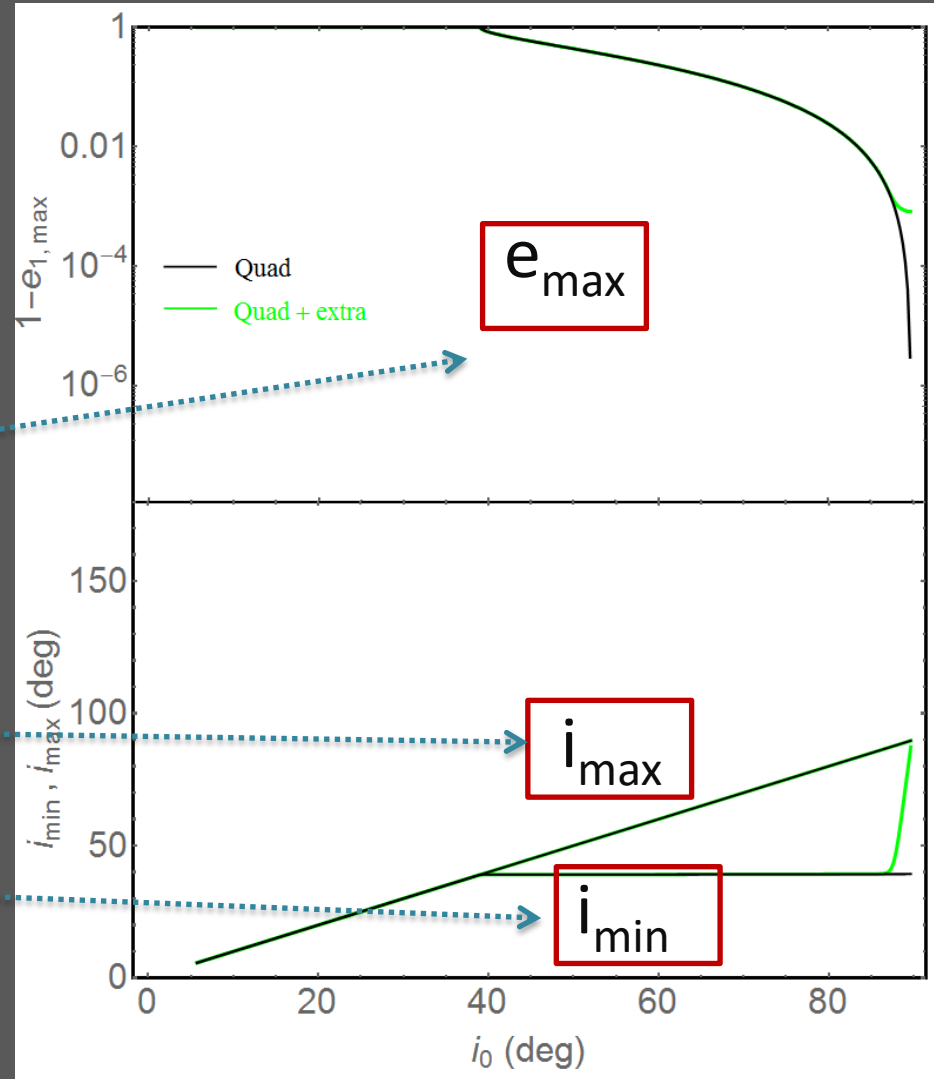
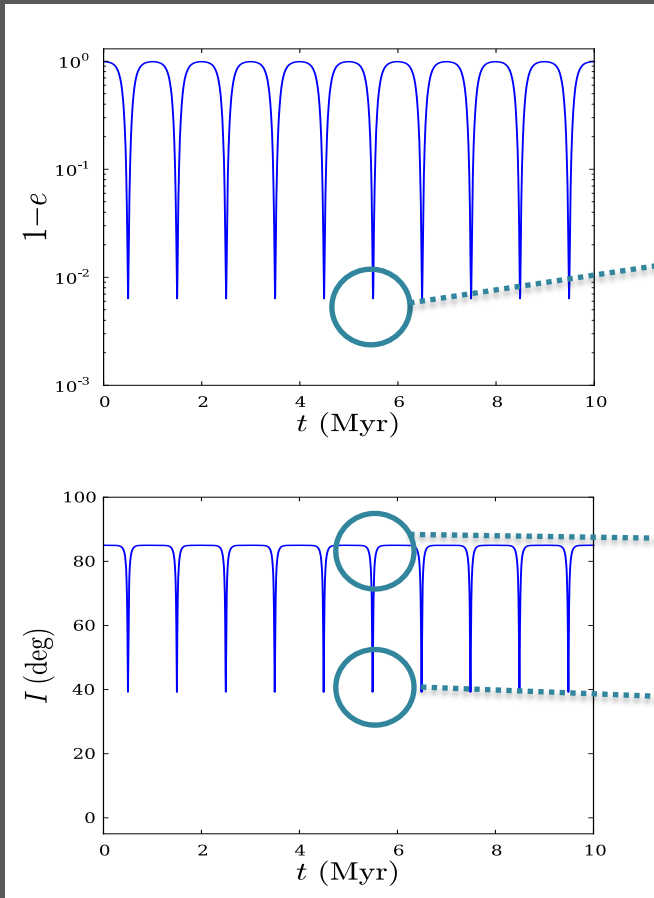
- (1)  $e_{\text{max}}$  is reduced/limited
- (2) flip is delayed/suppressed



# Parameter Survey

$m_0=1M_{\text{sun}}, m_1=1M_J, m_3=0.04M_{\text{sun}}, e_1=0.001, e_2=0.6$   
 $a_1=6 \text{ AU}, a_2=100 \text{ AU}$

- We setup each system by varying the **initial inclination**



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$m_0=1M_{\text{sun}}, m_1=1M_J, m_3=0.04M_{\text{sun}}, e_1=0.001, e_2=0.6$   
 $a_1=6 \text{ AU}, a_2=100 \text{ AU}$

■ Turn on Octupole →

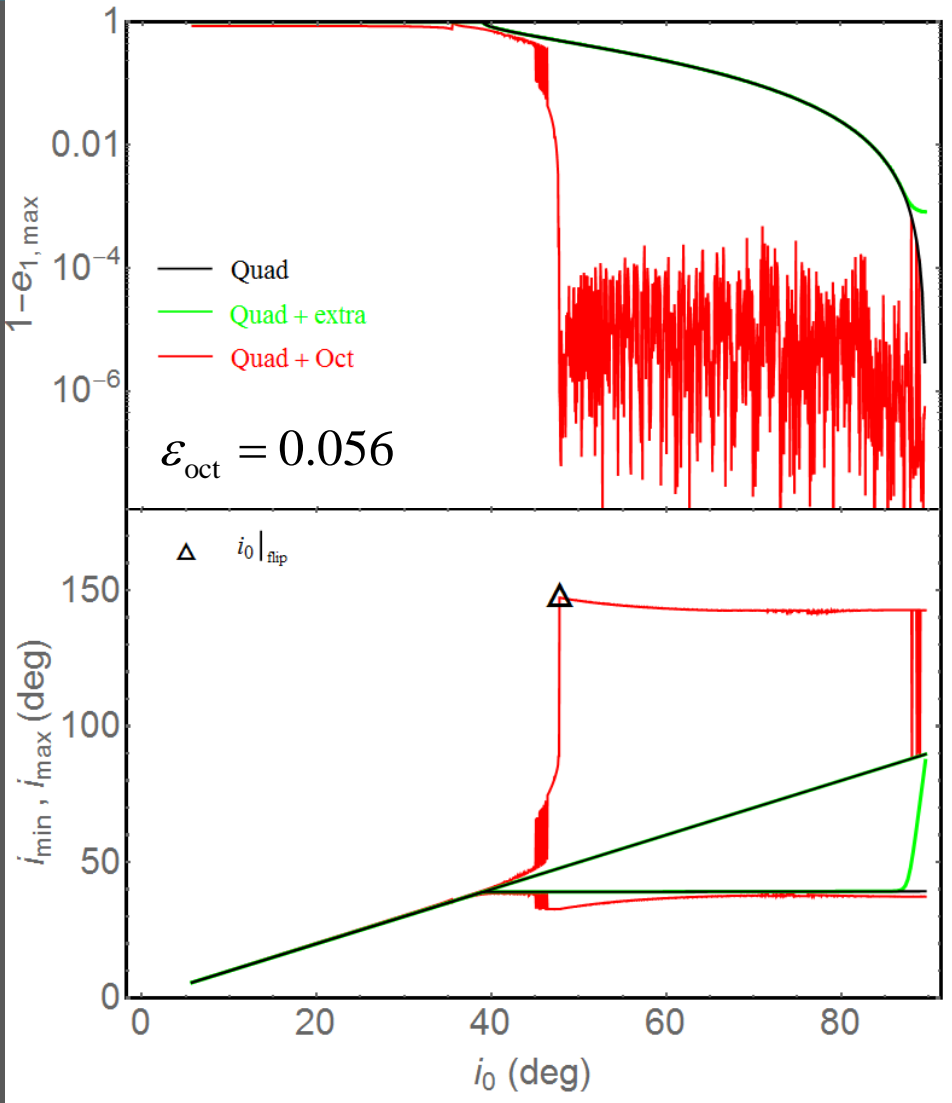
$$\epsilon_{\text{oct}} = \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

Strength of the octupole potential relative to the quadrupole

■ We find →

--- Very large  $e$  even for modest  $I$

---  $L_p$  can flip with respect to outer binary





# Suppression of extreme orbital evolution

Turn on SRF  $\rightarrow \mathcal{E}_{\text{SRF}}$

relative strength of the SRF potential

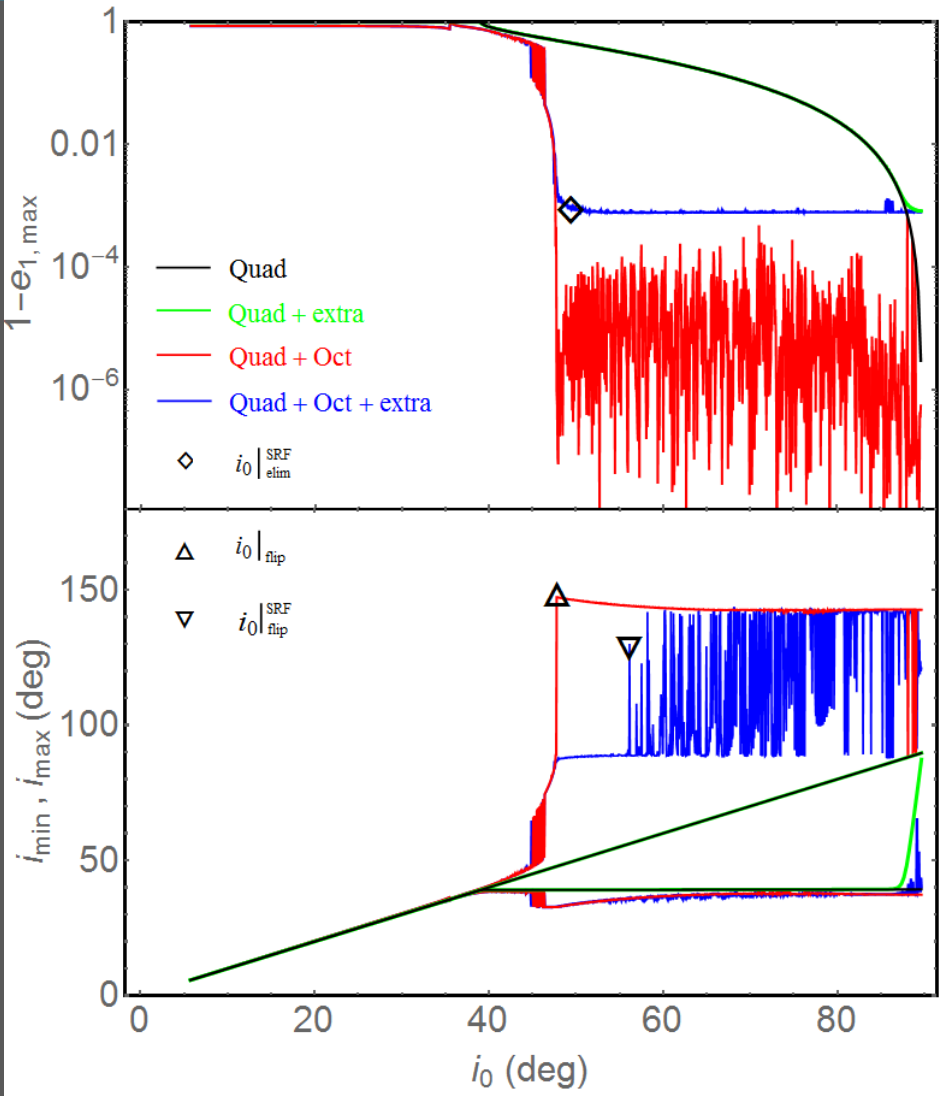
PRM.	$\mathcal{E}_{\text{Oct}}$	$\mathcal{E}_{\text{GR}}$	$\mathcal{E}_{\text{Tide}}$	$\dot{\omega}_{\text{GR}}/\dot{\omega}_{\text{K}}$	$\dot{\omega}_{\text{Tide}}/\dot{\omega}_{\text{K}}$
case	0.056	$2.93 \times 10^{-4}$	$1.32 \times 10^{-12}$	$6.86 \times 10^{-3}$	7.28

---  $e_{\text{max}}$  is reduced/limited

--- flip is delayed/suppressed

Limiting Eccentricity (analytic) regardless of octupole strength

$$\left[ \frac{\dot{\omega}_{\text{GR}}}{\dot{\omega}_{\text{K}}} + \frac{1}{15} \frac{\dot{\omega}_{\text{Tide}}}{\dot{\omega}_{\text{K}}} f(e_1) + \frac{1}{3} \frac{\dot{\omega}_{\text{Rot}}}{\dot{\omega}_{\text{K}}} \right]_{e_1=e_{\text{lim}}} = \frac{9}{8} e_{\text{lim}}^2$$



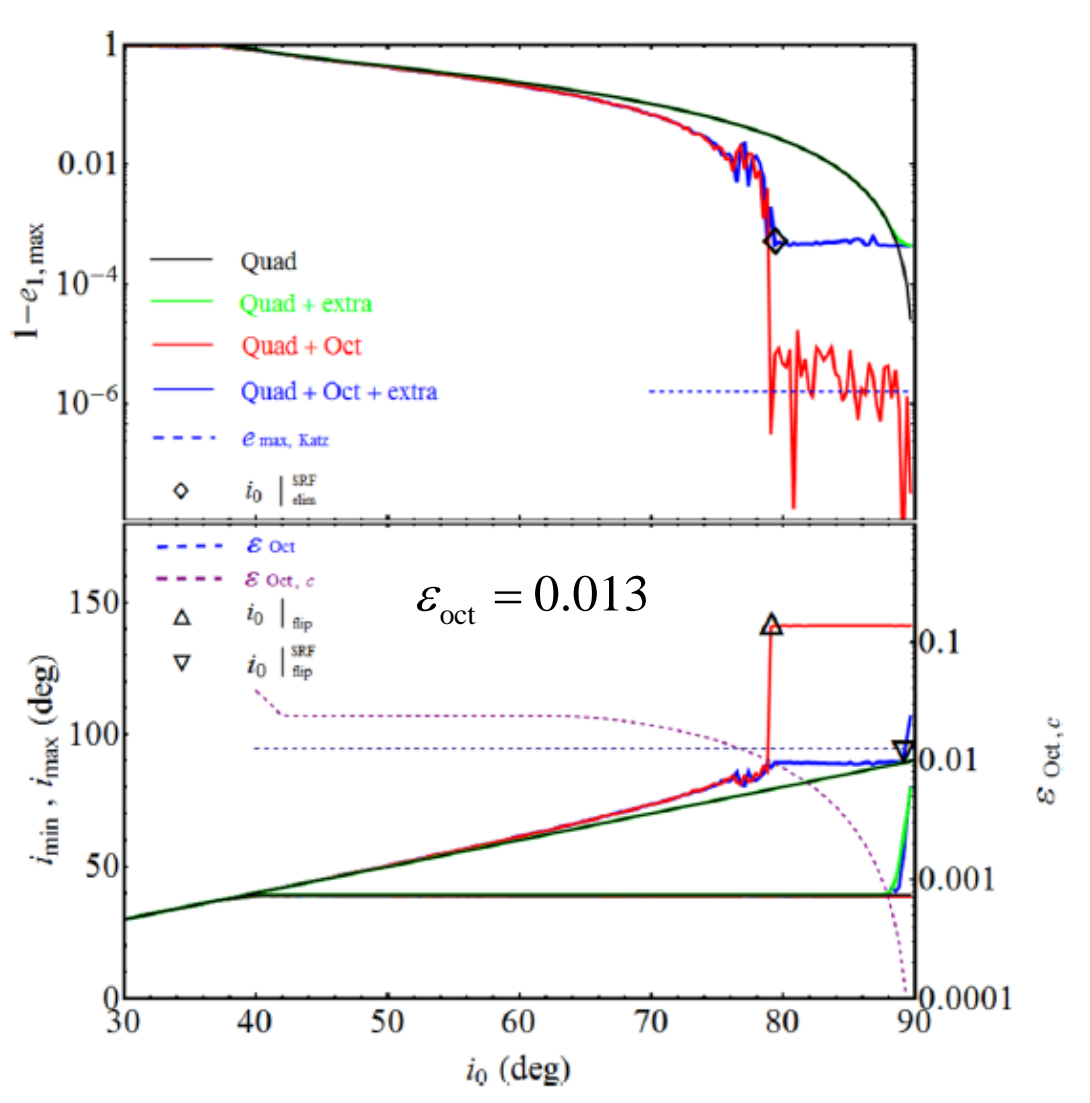
# Suppression of extreme orbital evolution

Smaller  $\mathcal{E}_{\text{Oct}}$

$\mathcal{E}_{\text{Oct}}$	$\mathcal{E}_{\text{GR}}$	$\mathcal{E}_{\text{Tide}}$	$\dot{\omega}_{\text{GR}}/\dot{\omega}_{\text{K}}$	$\dot{\omega}_{\text{Tide}}/\dot{\omega}_{\text{K}}$
0.013	$2.15 \times 10^{-5}$	$9.12 \times 10^{-14}$	$6.74 \times 10^{-4}$	7.32

--- window of influence  
(initial inclinations)  
become narrow

$m_0=1M_{\text{sun}}, m_1=1M_{\text{J}}, m_3=1M_{\text{sun}}$   
 $e_1=0.001, e_2=0.2$   
 $a_1=6 \text{ AU}, a_2=100 \text{ AU}$



Introduction

Oct (test mass)

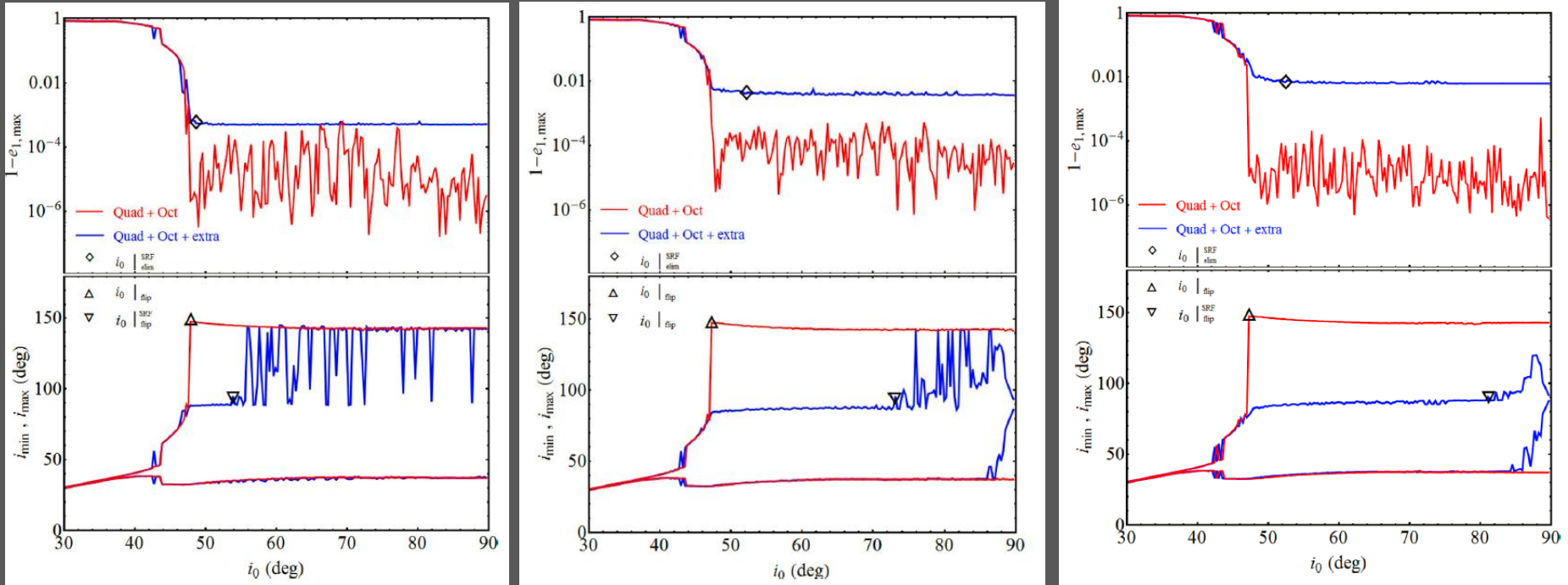
Summary

SRFs

Oct (comparable mass)

# Suppression of extreme orbital evolution

■ The relative strength of the SRF become stronger



$m_0=1M_{sun}, m_1=1M_J, e_1=0.001, e_2=0.8$

PRM.	$\mathcal{E}_{GR}$	$\mathcal{E}_{Tide}$	$m_3$	$a_1$	$a_2$
a	$3.96 \times 10^{-5}$	$1.79 \times 10^{-13}$	1	6	200
b	$2.37 \times 10^{-4}$	$1.39 \times 10^{-9}$	1	1	33.33
c	$2.37 \times 10^{-3}$	$1.39 \times 10^{-8}$	0.1	1	33.33

--- window of "flips" become narrow

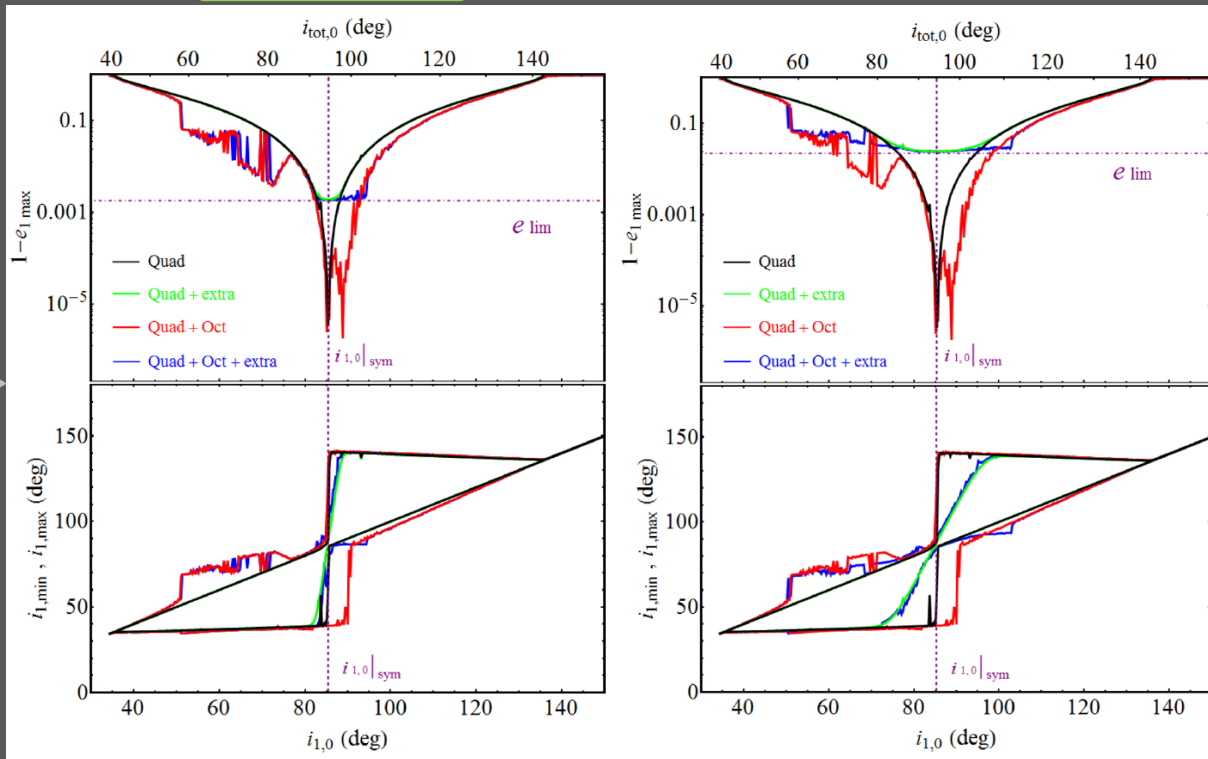
Summary

(comparable mass)

# Parameter Survey

## Comparable-mass cases: (BH/NS/WD/Main sequence star binary)

$\epsilon_{\text{Oct}}$	$\epsilon_{\text{GR}}$	$\epsilon_{\text{Tide}}$	$a_1$ (au)	$a_2$ (au)	$e_{2,0}$	$m_1$ ( $M_{\odot}$ )	$m_2$ ( $M_{\odot}$ )	$R_1$ ( $R_{\odot}$ )
0.022	$4.33 \times 10^{-5}$	$4.00 \times 10^{-11}$	1	10	0.5	0.5	1	0.5
0.022	$4.33 \times 10^{-5}$	$4.00 \times 10^{-6}$	1	10	0.5	0.5	1	5



$\epsilon_{\text{Oct}} = 0.022$

symmetry of eqs. is broken

Introduction

Oct (test mass)

Summary

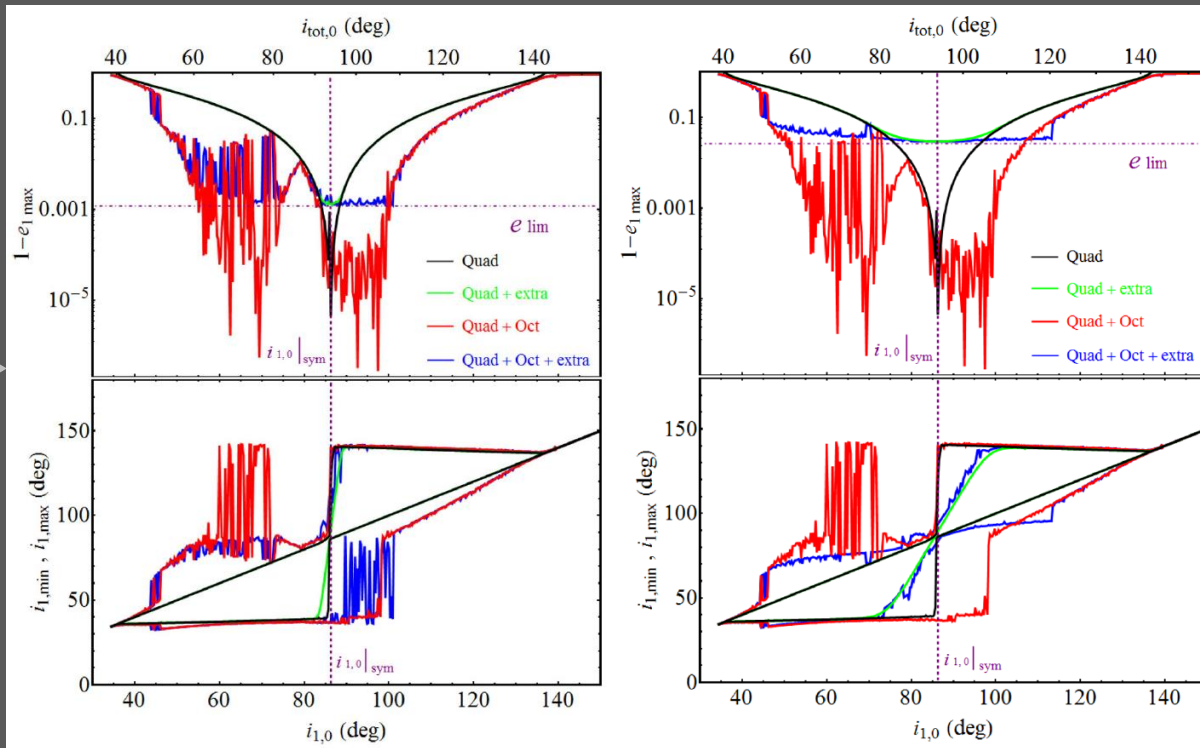
SRFs

Oct (comparable mass)

# Parameter Survey

## Comparable-mass cases: (BH/NS/WD/Main sequence star binary)

$\epsilon_{\text{Oct}}$	$\epsilon_{\text{GR}}$	$\epsilon_{\text{Tide}}$	$a_1(\text{au})$	$a_2(\text{au})$	$e_{2,0}$	$m_1(M_{\odot})$	$m_2(M_{\odot})$	$R_1(R_{\odot})$
0.042	$5.53 \times 10^{-5}$	$7.64 \times 10^{-12}$	1	12	0.6	0.3	0.8	0.3
0.042	$5.53 \times 10^{-5}$	$9.83 \times 10^{-6}$	1	12	0.6	0.3	0.8	5



$\epsilon_{\text{Oct}} = 0.042$

Introduction

Oct (test mass)

Summary

SRFs

Oct (comparable mass)



# Conclusions

- The main contribution of the oct. terms to eccentricity and inclination excitation
    - provides a “window of influence”, the width of which grows with  $\epsilon_{\text{oct}}$  (the importance of the octupole effects)
  - The SRFs can indeed affect the orbital evolution in the oct. order
    - impose a strict upper limit on the maximum achievable eccentricity (regardless of octupole strength)
- Combining the further studies in Kassandra R.A.+16,17, most of the properties of Oct. Kozai dynamics can be understood analytically
- With increasing strength of the SRFs, orbital flips are increasingly confined to the region close to  $I_0=90^\circ$  (depends on  $\epsilon_{\text{SRF}}$ )



